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THE

GYROSCOPE

AND SOME OF ITS TECHNICAL APPLICATIONS

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INTRODUCTION

The properties imparted to a body by its rapid rotation (to which it has been customary to apply the term gyroscopic properties since the time of Foucault, who constructed an instrument that he called a gyroscope) are becoming ever more widely used today in various fields of technology. In this book, the properties of the gyroscope will be explained in a manner within the reach of all, and on the basis of this explanation a brief elementary theory of a few of the most important applications of the gyroscope will be presented.

A fundamental study of gyroscope theory requires an acquaintance with higher mathematics and with theoretical mechanics (although only to the extent included in the programs of the higher technical schools). Our scientific textbook literature includes excellent manuals, some of them giving a detailed exposition of gyro theory (I might mention the excellent book by the late Academician A.N.Krylov "Obshchaya teoriya giroskopov i nekotrykh tekhn. ikh primeneniy" (General Theory of the Gyroscope and Some of Its Technical Applications) 2nd edition, 1936). In compiling the present booklet, however, the author set himself a different task: to provide brief information on the properties of the gyroscope for readers unacquainted with either higher mathematics or theoretical mechanics, but with a certain amount of experience in production, desiring to understand the mechanism of action of what are known as gyroscopic instruments, in which the properties of a rapidly rotating gyro find application.

The reading of the main text of this book (in large print) will occasion no

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difficulty for readers acquainted with the elementary mathematics and elementary physics as taught in the middle schools from which they graduated, while the supplementary material (in small print) may be useful reading for students with more extensive preparation (being familiar with trigonometry).

The author hopes that this book will be useful in the hands of mechanics working in gyroscopic instrument building, and also of a wide group of readers interested in mechanical problems. Those wishing to go more deeply into the study of gyroscope theory will find more extensive material in the above-mentioned book by Krylov.

Section 20 of this book, "Stability of a Rotating Projectile" was gone over in manuscript by Prof. L.G.Loytsinskiy, to whom I express deep gratitude for his valuable suggestions. I am likewise deeply grateful to V.K.Gol'tsman, who carefully read through the entire manuscript and made a number of valuable suggestions.

Ye.Nikolai

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CHAPTER I

PROPERTIES IMPARTED TO A BODY BY RAPID ROTATION

Section 1. The Rapidly Spinning Top and its Utilization in Technology

Who has not played with a top as a child? So long as its young owner has not placed it in rapid rotation, the top lies lifeless and motionless. But no sooner is it placed in rapid rotation than it comes to life and acquires remarkable properties. Who has not felt satisfaction in watching a rapidly spinning top maintain its equilibrium, balancing at the tip of its spindle, and watching how it quietly continues to spin, precisely supported by some invisible force (Fig.1)?
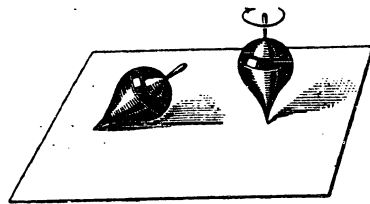
Fig.1

The surprising stability imparted to a top by rapid rotation has long attracted the attention of inquiring minds. About 200 years ago an attempt was made in the British Navy to utilize this property of a rapidly spinning top to provide a stable "artificial horizon" on shipboard, capable of replacing, in fog, the visible horizon required by the mariner for his astronomical observations. The shipwreck of the frigate "Victory", on which this instrument was being tested (the inventor of the "arti-

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ficial horizon", Serson, was lost in this disaster) put an end to this attempt.

During the next century no new attempts at a practical utilization of the spinning top were made. A new impetus in this direction was given by the famous experiments of Foucault, reported to the Paris Academy of Sciences in 1852. Among other experiments, Foucault demonstrated the instrument constructed by him, called a "gyroscope", whose primary component consisted of a rapidly rotating rotor (top) and which, for the first time, provided a direct laboratory demonstration of the diurnal rotation of the earth. The term "gyroscope" (in literal translation, "instrument exhibiting rotation") has been maintained in the scientific world. Today this term is used, in the broadest sense, to denote any instrument in which the peculiar properties of a body in rapid rotation are utilized; these properties are commonly called gyroscopic properties.

In the same famous report of 1852, Foucault showed that it was possible (at least theoretically) to construct a gyroscopic instrument to determine the position of the meridian (North-South direction) at a given place. Thus was expressed, for the first time, the idea of a mechanical (nonmagnetic) compass, constructed on the principle of the gyroscope, and capable of completely replacing the magnetic compass. The problem of replacing the magnetic compass by a mechanical one had become particularly urgent with the appearance of large masses of iron on board warships, and in connection with the increasing complexity of the electric equipment of these ships, which interfered with operation of the magnetic compass on them. However, there were immense difficulties in the way of any realization of Foucault's idea; these were surmounted only fifty years later, at the threshold of the present Century. The exceptional progress in technology made it possible for highly developed gyro compasses to appear at the beginning of the Twentieth Century, to attain general recognition, and widespread use in the navies of the whole world.

Today gyroscopic instruments are gaining ever increasing importance in various fields of technology. Military and naval technology is equipped with a large number

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of instruments based on the gyroscope principle. The gyroscope has found particular-
ly widespread use in aviation. Confident blind flying in the absence of visible
landmarks, and prolonged distance flights for many hours, without landing, have be-
come possible, owing to the large number of gyroscopic aviation instruments with
which modern aircraft is equipped.

Who has not mused, in childhood years and perhaps even at a more mature age, on
the question of the cause of the surprising behavior of a rapidly spinning top?
What is the explanation of the remarkable phenomena observed during the rapid rota-
tion of bodies, phenomena to which we give the collective term of gyroscopic phenome-
na? In this book we will try to answer these questions. We will also show how gyro-
scopic phenomena have been utilized for various purposes in modern gyroscopic instru-
ments and installations.

Section 2. The Gyroscope in Cardanic Suspension. Stability of the

Axis of a Balanced Gyro Imparted to it by Rapid Rotation

of the Gyro

Before beginning our explanation, let us say a few words on the simplest gyro-
scopic instrument, the gyroscope in a Cardanic suspension, which is the most impor-
tant component of most of the existing gyroscopic devices.

The rotor or top P is suspended in two rings A and B, constituting the Cardanic
suspension (Fig. 2). The outer ring of the suspension A rotates freely about its
vertical diameter ab, which is held in a fixed position. The inner ring B rotates
about the horizontal diameter cd of the outer ring A. This inner ring bears the axis
of rotation of the rotor P, which axis is perpendicular to the axis of rotation cd
of the inner ring B. Thus, this device comprises three axes of rotation which inter-
sect each other at one point 0: 1) the axis of rotation ab of the outer ring of the
suspension; 2) the axis of rotation cd of the inner ring; 3) the axis of rotation
ef of the gyroscope rotor. The rotation of the rotor gyroscope P about the axis ef

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is customarily termed the proper rotation of the gyroscope (in distinction to the rotations of the outer and inner rings of suspension). In accordance with this, let us term the axis ef "the axis of proper rotation" or simply the "gyro axis". The axes ab and cd are termed the Cardanic axes.
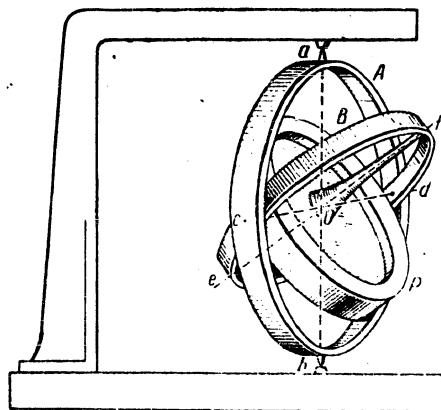
Fig.2

Various organizations in the USSR manufacture small school models of the gyroscope in Cardanic suspension for school physical laboratories; their price is modest. Everyone interested in gyroscopic phenomena who wishes to familiarize himself more closely with them is advised to acquire a small model of the gyroscope in Cardanic suspension. Such a model will enable the thoughtful observer to make many interesting experiments in which the essential nature and significance of these interesting phenomena will be disclosed.

We assume that we have at our disposal a model gyroscope in an Cardanic suspension and start an experiment to interpret the phenomenon of stability imparted to

the axis of the balanced* or free gyroscope by the rapid rotation of its rotor.

First of all let us explain what we mean by the term "balanced" or free gyro-scope. We shall term a gyroscope in a Cardanic suspension a free gyroscope if the
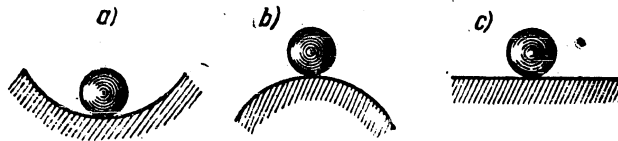
Fig.3

common center of gravity of the moving parts of the instrument, the rotor and the two rings, coincides with the point of intersection O of the three axes of rotation of the instrument; the axis of proper rotation ef and the two Cardanic axes ab and cd (Fig.2)**. Such a gyroscope maintains its equilibrium at any position of its rotor, and it is for this reason that it is termed free or, sometimes, astatic. This readily indicates that the equilibrium of the gyro in any of its positions must be considered neutral***, and a light tap on one of the gimbal rings is sufficient to bring the instrument out of its assigned position, to which it will not return, but;

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* Translator's note: In U.S. terminology, this would be a "free" gyro.
** Commercial models of gyroscopes in Cardanic suspension satisfy this condition with sufficient accuracy.
*** Stable, unstable and neutral equilibria are distinguished. A ball on a concave spher-ical surface (Fig.3a) is in stable equilibrium. Conversely, its equilibrium on a convex spherical surface (Fig.3b) is unstable. A sphere on a horizontal plane (Fig.3c) is in neutral equilibrium.

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executing a more or less marked deviation, will remain in some new equilibrium position*. The gyroscope has no stability at all while its rotor is not in rapid rotation.

The situation changes completely if we first impart a rapid natural rotation to the rotor of the gyroscope (by means of a thread or string wound around its axis). We now give the gyroscope any arbitrary position and then tap one of the gimbal rings. It can be felt immediately that, under the influence of the rapid rotation of the rotor, the gyroscope has acquired the peculiar property of energetically resisting the action of forces tending to change the direction of its axis. Under the action of the applied shock, the axis of the gyroscope (we mean the axis of the rotor) does not markedly change its direction, and close observation will show only slight and very rapid vibrations of the axis (which are termed nutational oscillations). This gives the impression that the rapid rotation of the rotor has imparted to the whole instrument some rigidity, a certain resistance to the action of the applied tap. We conclude that the rapid rotation of the rotor imparts a peculiar stability to the axis of a balanced gyroscope.

To make this experimental result convincing, the rotor of the gyro must be given as rapid a rotation as possible**.

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* The instrument, set in motion by a push, finally stops under the action of the forces of friction that are unavoidable in any instrument. If there were no friction in the instrument (and likewise no air resistance) then, if once set in motion by an impulse applied to one of the gimbal rings, the instrument would continue its motion for an indefinitely long period.

** For this purpose, a strong thread or string is wound around the axis of the rotor, pulling it first by its end, at a relatively slow speed and not very rapidly but increasing this speed gradually until the maximum speed of which the hand is capable is reached.

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The faster this rotation, the more distinctly will the stability acquired by the gyro axis be manifested.

It must be borne in mind that there is one position of the gyro in which it looses its property of stability on rapid rotation. This is the position when the axis of the rotor ef coincides with the axis of rotation ab of the outer gimbal ring (Fig.2). This is why the designers of gyro instruments take measures to make it impossible for the axis of the rotor to coincide with the axis of rotation of the outer ring of the Cardanic suspension.

Section 3. Stability of Rapid Rectilinear Motion. Jets Issuing Under High Pressure.
The Law of Inertia. Newton's Second Law of Motion.

What is the cause of this stability, of this seeming rigidity which is acquired by a gyroscope on rapid rotation of its rotor? In order to analyze this question, let us begin with the simplest cases of all.

From everyday experience, we know that the softest and most yielding bodies acquire an apparent rigidity on rapid motion. A jet of water forced from a fire-hose nozzle under high pressure is a very rigid body, which differs little in rigidity from a metal bar. Such a jet can easily knock a man off his feet. The higher the velocity of the individual particles of water forming the jet, the greater the resistance of the jet to all forces tending to change its direction, and the higher the pressure exerted by it on any obstacle encountered in its path.

A skater gliding rapidly over the ice is well aware that, the faster his motion, the more difficult will it be for him to suddenly change the direction of that motion. In the same way, the driver of a rapidly moving automobile knows that the faster he is driving, the more difficult will it be to avoid any obstacle suddenly appearing in his way, such as the form of an absent-minded pedestrian carelessly walking in the middle of the road.

In all these cases we have to do with manifestations of the law of inertia.

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This law, first discovered by Galileo, states that a moving body tends to maintain the direction of its motion and the magnitude of its velocity.

The inertia of a moving body is the cause of the above-mentioned phenomena, in which the stability of the rectilinear motion of a body at high speed is expressed. To elucidate the influence of the rate of motion still more clearly in these phenomena, let us perform a brief calculation.
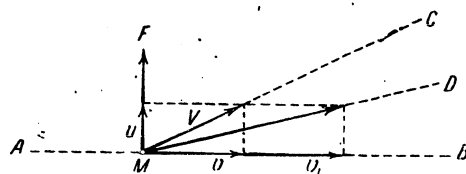
Fig.4

Let the sphere M be rolling with a velocity v along the horizontal plane of a table in the direction AB (Fig.4)*. Let us strike the sphere in a direction coinciding with the horizontal plane of the table and perpendicular to AB. The action of the shock will be expressed in the action of the force F on the sphere M during the negligibly short time interval τ in the direction of the shock, which is perpendicular to AB. What will be the effect of the action of this force?

Every force applied to a moving body causes a change in its velocity. This change in velocity is defined by Newton's Second Law of Motion which reads: A change in velocity u, caused by the force F acting during the period of a negligibly small time interval τ, has the direction of the force F, and its magnitude is proportional to the force F and to the time of its action τ, and is inversely proportional to the mass of the body m. This law is expressed by the formula

$$u = \frac{F}{m} \qquad (1)$$

* The velocity v is shown by the arrow pointing in the direction of the motion.

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On having acquired the velocity u in the direction of F under the action of the force F (i.e., perpendicular to AB), our sphere will then continue to move with the velocity V, which we obtain by combining the velocities v and u. This combining of velocities must be performed by the parallelogram rule (in the same way as the composition of forces). Construct a parallelogram on the segments v and u (in this case it will be a rectangle) and draw its diagonal V (Fig.4). This velocity V is the motion with which the sphere will continue. Obviously, as a result of the applied shock, the sphere will change the direction of its motion; it will now move in the direction MC, deviating from its original direction MB by the angle BMC.

Let us now assume that the original velocity of our sphere was not v, but twice as large; let us denote it by $v_1 = 2v$ (Fig.4). On repeating the same construction we find that now, under the action of the same shock of the impact force, the sphere, assuming the same additional velocity u, will deviate from its original direction by the smaller angle BMD (Fig.4); in general, the higher the velocity possessed by the sphere before the impact, the smaller will be the angle, other conditions being equal, by which it will deviate from its original direction after the impact. It is this that we have in mind when we say that the rectilinear motion of a body is more "stable", the higher its velocity*.

The manifestations of stability of rapid rectilinear motion comprise all the facts described at the beginning of this Section.

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* The term "stability of motion" has various meanings in science. In using this term, it is necessary to indicate precisely the sense in which it is to be applied. We emphasize once again that here, in speaking of the stability of a rapid rectilinear motion, we mean that the higher the velocity of the body, the smaller will be the angle by which the body will deviate from its orginal direction under the action of a lateral impact of given intensity, and the less will the body obey, in this sense, the action of a lateral impact.

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The law of inertia and Newton's Second Law of Motion give a complete explanation of these phenomena.

Section 4. Inadequate Explanation of the Stability of a Rapidly Rotating Gyro. Degrees of Freedom of a Gyro. Loss of Stability by a Rapidly Rotating Gyro with Less Degrees of Freedom.

Let us now return to the question of the stability of a rapidly rotating gyro.

We have seen that the stability of rapid rectilinear motion finds its explanation in the law of inertia. At first glance, it would seem that the explanation of the stability of a rapidly rotating gyro may also be found in this same law.

As it is inherent in a body in rectilinear motion to maintain the direction of its motion, so it is inherent in the rotating rotor of a gyroscope to maintain the plane of its rotation (thereby making it an inherent property of the axis of the rotor to maintain its direction). The higher the speed of rectilinear motion, the more energetic is the resistance of the body to forces tending to vary the direction of its motion. Similarly, the more rapidly the rotor of a gyro rotates, the more energetically will it resist forces tending to vary the plane of its rotation or the direction of its axis.

These are proved facts, and such an explanation appears entirely acceptible at first glance. However, the obvious inadequacy of this explanation must be emphasized at this point, for it does not cover the essence of the matter and leaves undisclosed the most substantial and important features of the question with which we are concerned.

The reader will be able to convince himself of the fact that this explanation is inadequate by making the following simple experiment:

Let us take our model of the gyro in Cardanic suspension, and set the rotor of the gyro in rapid proper motion. Firmly hold the outer gimbal ring, preventing it from rotating about its axis, and at the same time tap the inner ring. Surprisingly,

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we now find the stability of our gyroscope has entirely disappeared. No matter how rapidly the rotor of the gyro is now rotating, it will be unable to resist forces tending to vary the direction of its axis, just as though no rotation at all had been imparted to it.

What has changed by comparison with the condition of the experiments described in Section 2? Only the fact that, in the former case, the outer ring of the Cardanic suspension was able to rotate freely about its axis while now we have fixed this ring and destroyed its free rotation. It is clear that destruction of the freedom of rotation of the outer gimbal ring completely deprives a rapidly rotating gyro of its stability, and completely deprives it of its ability to resist the action of forces tending to vary the direction of its axis.

From the point of view of the considerations presented above, this fact (now discovered) of the loss of stability of the gyro when the outer gimbal ring is fixed remains entirely incomprehensible. Obviously, the above explanation of the stability of a gyro does, in fact, evade the essence of the question, and must definitely be recognized as inadequate and unsatisfactory.

We will return again to this explanation of the loss of stability of a gyroscope when the outer ring of the Cardanic suspension is fixed. For the time being, however, we content ourselves with the following remark:

In the gyro described in Section 2, there are three axes of rotation, corresponding to the three possible rotations of the instrument: rotation of the outer ring, rotation of the inner ring, and proper rotation of the gyro rotor itself. Accordingly, a gyroscope with this arrangement is called a gyroscope with three degrees of freedom.

By fixing the outer ring of the suspension, we destroy one of the degrees of freedom of the gyro, corresponding to the rotation of the outer ring about its axis. For this reason, a gyro with a fixed outer ring is termed a gyroscope with two degrees of freedom.

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We may now formulate what has been established in Section 2 and in the present Section in the following way: a gyroscope with three degrees of freedom has the property of stability in rapid rotation; this property is entirely lost by a gyroscope with two degrees of freedom*.

We also note that the experiments described in this Section will give a convincing result only if we actually deprive our gyro of one of its degrees of freedom, i.e., if we firmly hold the outer ring of the suspension and do not allow it to rotate about its axis. The slightest freedom of rotation left to the outer ring will result in certain stability - although a weakened one - of the axis of the gyro rotor.

Section 5. Action of Forces Applied to the Axis of a Rapidly Rotating Gyro.

Let us now proceed to the explanation of the stability of a rapidly rotating gyro with three degrees of freedom, and of the instability of a gyro with two degrees of freedom.

In Section 3, when explaining the stability of rapid rectilinear motion, we considered the action of a transverse impact force on a body in rectilinear motion. We shall proceed similarly here. Let us define the result of the action of lateral forces applied to the axis of a rapidly rotating gyro. The answer to this question will give a clue to the explanation of all gyroscopic phenomena.

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* We remark that the gyro with three degrees of freedom may likewise be transformed into a gyro with two degrees of freedom by destroying the degree of freedom corresponding to the rotation of the inner ring of the suspension about its own axis. This may be done by rigidly attaching the inner ring at a right angle to the outer ring and leaving the outer ring free. It is obvious that, in this case, the gyro, deprived of one of its degrees of freedom, will lose its stability and rapid rotation.

First we define the deviation of the axis of a rapidly rotating gyro from its original position, under the action of given transverse forces applied to the gyro axis. Of course, this question may also be posed differently. One might ask what transverse forces must be applied to the axis of a rapidly rotating gyro in order to produce a given deviation from its original direction. This inverse formulation of the question is more convenient for us. Let us now consider it.

Assume that the rotor of the gyroscope ABCD is given a rapid proper rotation about its axis KL, which we assume to be directed, e.g., horizontally (Fig.5; the gimbal rings are not shown on this diagram). The rotation of the rotor about the axis KL is assumed to be directed as shown in the diagram by the curved arrow, i.e., we assume it to be directed clockwise, if viewed from the end L of the axis. All points of the circumference of the rotor have the same velocity v_2 directed along the tangents to this circle. Figure 5 shows the velocities of the points A, B, C, D, lying at the ends of the horizontal and vertical diameters of the rotor.
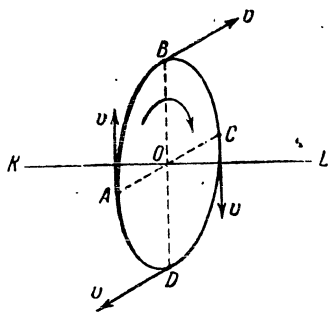
Fig.5

We now assume that we vary the direction of the axis of the rotor KL, by turning it through the small angle in the horizontal plane (Fig.6); the new position of the axis KL will be denoted by K_1L_1. In this case, the plane of the rotor ABCD is turned through the same angle about the vertical line zz and takes the position A_1BC_1D. We assume that the rotation of the axis of the rotor through the angle τ takes place during the course of the short time interval α. The question is what forces must be applied to the axis KL to the rotor to effect a rotation of this axis.

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Let us try to ascertain how the rotation of the rotor plane about the straight line zz through the angle α will affect the velocities of the individual points on the circumference of the rotor.

It is obvious that the velocities of the points A and C (which, after the rotation, will occupy the positions A_1 and C_1) will preserve both magnitude and direction
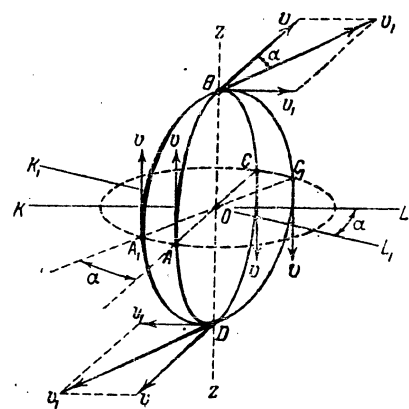
Fig.6

throughout this rotation (Fig.6). The situation at the points B and D will be different; here, while the magnitude of the velocity likewise remains unchanged, its direction does vary.

Let us take the point B. Before the rotation, its velocity v was directed along the tangent to the circumference ABCD. After the rotation, the velocity v_1 of the point B, numerically equal to v ($v_1v_1 - v$), is directed along the tangent to the circumference A_1BC_1D; in the time τ it will rotate through the angle α, together with the entire plane of the rotor, about the line zz.

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Figure 7 shows the direction of the velocities v and v_1 in projection onto the plane AA_1 CC_1, perpendicular to the line zz.

Let us construct a parallelogram at point B in which the segment v_1 is the diagonal and the segment v is one of the sides; the second sides of this parallelogram will be termed u_1 (Figs.6 and 7). It is easy to see that the transition from the velocity v of point B to its new velocity v_1 is equivalent to the appearance, at this point, of a new component of velocity u_1 which, on combining with the previous velocity v, gives the new velocity v_1 of the point B. In view of the smallness of the angle α assumed by us, it may be considered that the magnitude of the additional velocity component u_1 is equal to $v\,\alpha$, i. e., $u_1 = v\,\alpha$ *, the direction of the velocity u_1 may be considered perpendicular to the plane ABCD or parallel to the axis of the rotor KL. The velocity component that appears at the point D will be the same in magnitude but opposite in direction.
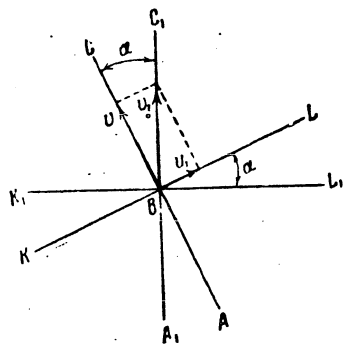
Fig.7

However, this is not all. Let us return to the point B. It must be taken into consideration that during the time τ the point B, participating in the rotation of

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* When the angle α is small, the second side u_1 of the parallelogram may be considered to be equal to the arc of a circumference of a radius v corresponding to the central angle α; however, the arc is equal to the product of the radius and the central angle. Consequently, $u_1 = v\,\alpha$.

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the rotor about its axis KL, travels the short path $BB_1 = v\tau$ along the circumference ABCD, and by the end of the interval of time will no longer be on the line zz but at a distance PB_1 from this line (Fig. 8); in this case, owing to the rotation of the rotor plane about the line zz, our point will also have acquired a new velocity (denoted by u_2) perpendicular to the plane ABCD (i.e., parallel to the rotor axis KL) and equal to the product of the distance PB_1 of the point B_1 from the line zz and the angular velocity of rotation of the rotor plane about the line zz. Since this angular velocity (denoted by w_1) is equal to the ratio of the angle of rotation to the time (i.e., $w_1 = \dfrac{\alpha}{\tau}$), we conclude that

$$u_2 = PB_1 \, \omega_1 = v\tau\frac{\alpha}{\tau} = v\alpha$$
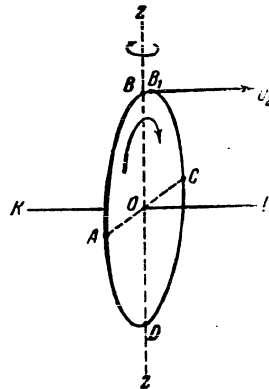
Thus, the above considerations indicate that, at the point B (or, more exactly, at the point of the circumference of the rotor which was located at B at the beginning of the time interval τ and at B_1 at that time interval), there appears a second velocity component u_2, equal and parallel to the velocity u_1. On compounding the velocities u_1 and u_2, we conclude that the rotation of the rotor plane through the angle α is accompanied by the appearance at the point B of a corresponding velocity u, directed parallel to the axis of the rotor KL and equal to

$$u = u_1 + u_2 = v\alpha + v\alpha = 2\,v\alpha$$

A velocity, equal in magnitude but opposite in direction, also appears at point D.

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As for the remaining points of the circumference ABCD, similar arguments (whose details will not be given here) lead to the conclusion that, at the points of the semicircumference ABC, there appear velocity components parallel to the rotor axis KL, which are numerically less than u and have the same direction as the velocity u at point B (Fig.9). Velocities that are the same in magnitude but opposite in direction appear at the points of the semicircumference ABC.

After having defined the mechanism of action of the rotation of the rotor plane on the velocities of its individual points, we may now proceed to our main problem which is to define the forces that must be applied to the rotor axis to cause its plane to rotate through the angle α.

Let us return to Newton's Second Law of Motion. We know that, if a moving body of mass m acquires, during the time τ, a change in velocity u, then this change in velocity is the result of the action of a force F, applied to the body and having the direction of the change in velocity u, being equal to

$$F = m \frac{u}{\tau}$$

(cf. eq. (1) in Section 3).

Let us assume all the mass of the rotor to be concentrated on its circumference ABCD, and let us divide this total mass into individual particles of mass m. Let us apply Newton's Second Law of Motion to the particle at point B. On rotation of the

Fig.9

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rotor plane through the angle α during the time τ , this particle will acquire the change in velocity u = 2 v α , directed parallel to the rotor axis KL (Fig.9). We
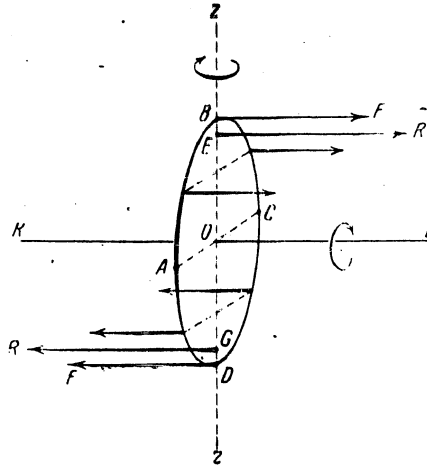
Fig.10

must conclude that this particle, during the time τ , is subjected to the action of the force F, likewise directed parallel to the rotor axis KL (Fig.10) and equal to

$$F = m \frac{u}{\tau} = 2\ mv\ \frac{\alpha}{\tau}$$

The same force, but opposite in direction, is applied to the point D. To the remaining points of the circumference ABCD are applied forces less than F; and at the points A and C these forces are equal to zero.

All the forces applied to the points of the semicircumference ABC are compounded into a single resultant R, directed in the same way as all these forces, i.e., parallel to the rotor axis KL on the side of the end L of this axis. The same resultant,

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but opposite in direction, is obtained by combining all component forces applied to the points of the semicircumference ADC.

The two forces R_1, equal and parallel but directed toward opposite sides and applied at the points E and G, form what is termed a couple of forces. The segment EG is termed the arm of the couple, while the product $R \cdot EG$ is the moment of the couple. Let the moment of the couple so obtained be denoted by the letter M and let EG = d. Then, M = Rd.

Let us denote the angular velocity of the proper rotation of the rotor about the axis KL by the symbol w . A calculation, whose details will not be given here, leads to the following expression for the moment M of the couple of forces so obtained:

$$M = Jw \frac{\alpha}{\tau}$$

or

$$M = Jw\, w_1 \tag{2}$$

if, as above, we denote

$$w_1 = \frac{\alpha}{\tau}$$

i.e., if the symbol w_1 denotes the angular velocity of rotation about the axis zz. This moment M is called the gyroscopic moment.

Here J is a constant factor depending on the mass, shape, and dimensions of the rotor; it is termed the moment of inertia of the rotor*.

Thus, the rotation of the rotor plane through the angle α about the line zz assumes the existence of a couple of forces with a moment M. The forces R forming this couple are applied at the points E and G, and tend to rotate the plane of the rotor

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* We will return in Section 13 to the derivation of eq. (2).

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about the line AC (Fig.10). The effect of the action of this couple remains un-
changed if we assume that the forces forming it are applied not at the points E and
G, but at any points E_1 and G_1 on the rotor axis KL (Fig.11). The magnitude of the

forces R_1 of this couple and the
magnitude of its arm $E_1 G_1 = d_1$
may be taken arbitrarily; it is
important only that the condition

$$R_1 d_1 = M = J \dot{w} \ w_1$$

be satisfied.

Let us finally sum up. We
pose the question what forces must

Fig.11   be applied to the rotor axis KL,
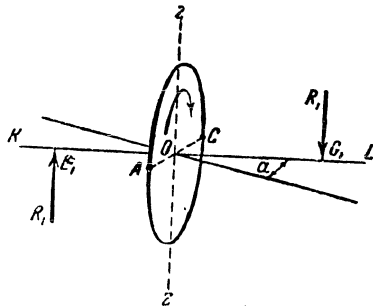
in order to rotate this axis in the horizontal plane through the angle α . We see
now that, for this purpose, it is necessary to apply to the rotor axis the couple of
vertical forces R_1 of moment

$$M = Jw \ \frac{\alpha}{\tau}$$

This result cannot be otherwise than unexpected. To turn the axis of the rotor
in a horizontal plane it is necessary to apply to this axis a couple of forces in a
vertical rather than in a horizontal direction. If the rotor were not rotating a-
bout the axis KL, then, under the action of the vertical forces R_1 applied at the
points E_1 and G_1, it would of course rotate about the horizontal line AC. In the
presence of the proper rotation of the rotor about the axis KL, the same forces pro-
duce a rotation of the rotor about the vertical line zz instead.

This fact is the key to the explanation of all the surprising properties of a

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rapidly rotating gyroscope, to be discussed below.

Section 6. The Rule of Precession

Let us return again to the result obtained in the preceding Section.

We have seen that a couple of forces R_1, applied to the axis of a rapidly rotating rotor KL, causes a rotation of that axis in the horizontal plane through the angle α (Fig.11). If the angular velocity of the proper rotation of the rotor about the axis KL is equal to w , and if the applied couple of forces acts during the short time interval τ , then the angle of rotation α is determined by the equation

$$J \omega \cdot \frac{\alpha}{\tau} = M$$

where J is the moment of inertia of the rotor and M is the moment of the applied couple; hence

$$\alpha = \frac{M \cdot \tau}{J \cdot \omega}$$
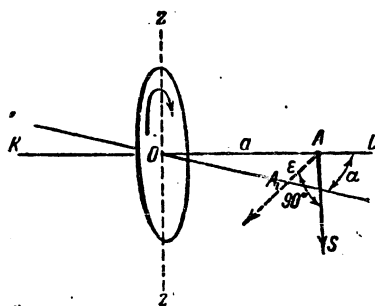
Fig.12

Let us now assume that the rotor axis is subjected, not to a couple of forces but to the single transverse force S, which acts during the short time interval perpendicular to the axis KL (Fig.12); the direction of the force S is assumed to be vertical. What will be the result of the action of the force S?

The design of the instrument is such that the point O of the axis KL, the point of intersection of the Cardanic axes, must remain fixed. The fixing of this point has the consequence that when the force S is applied at the point A of the axis KL (Fig.12), then the force of reaction S_1 of this fixed point equal in magnitude to the force S but opposite in direction, appears at that point O. The forces S and S_1

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form a couple of forces. Thus, we have a couple of forces applied to the axis KL, with the distance a of the point of application A of the force S from the fixed point O being the arm of this couple and its moment being equal to M = Sa.

The result of the action of this couple of forces is already known to us. It will cause, during the time τ , the rotation of the rotor axis KL in the horizontal plane through the angle α (as shown in Fig.12). The value of the angle α is determined by the formula

$$\alpha = \frac{M\tau}{J\omega} = \frac{S\,a\tau}{J\omega}$$

This will be the result of the action of the force S applied to the point A.

Thus, the result of the action of the force S applied to the axis of the rapidly rotating gyro is the rotation of this axis through the angle α in a plane perpendicular to the direction of the force S. This phenomenon is termed the precession of the gyroscope. Under the action of the force S, the axis of the gyroscope is said to "depart" from its assigned direction, or to undergo "precession" in the plane perpendicular to the direction of the force S.

When the axis KL is rotated through the angle α , the point A, at which the force S is applied, is displaced to the position A_1 (Fig.12). Let us denote by E this displacement or "departure" AA_1 of the point A and let us mark the direction of this segment by a dotted arrow. The diagram (Fig.12) indicates clearly that the direction of the displacement E is obtained if we rotate the direction of the force S through 90° about the axis KL in the direction in which the rotor of the gyroscope is rotating about this axis.

To sum up all above statements, we arrived at the following conclusion, which we will term the rule of precession (or the law of precession) of the gyroscope:

Under the action of the force S applied to the axis of a rapidly rotating gyroscope in a direction perpendicular to this axis, the axis of the gyro undergoes

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precession in a plane perpendicular to the direction of the force.

The direction of displacement of the point A at which the force S is applied is found by rotating the direction of the force S through 90° about the gyro axis in the direction in which the gyro rotor is rotating about this axis.

The magnitude of the angle of deviation α of the gyro axis from its original direction is determined by the formula

$$\alpha = \frac{Sa\,\tau}{J\,w} \tag{3}$$

where a is the distance of the point of application of the force S from the fixed point O; J is the moment of inertia of the rotor; w is the angular velocity of its proper motion; and τ is the time of action of the force S, which time is assumed to be short.

This law of precession gives the key to the explanation of all gyroscopic phenomena and to the construction of a theory of gyroscopic instruments.

In our derivation we assumed the force S to be directed perpendicular to the rotor axis KL. We assume now that the force S applied to the rotor axis KL (which we assume, as before, to be horizontal), is directed in the vertical plane containing the axis KL, but not perpendicular to the axis KL (Fig.13). It is not difficult to calculate the effect of the action of the force S.

Let us resolve the force S by the parallelogram rule into the components S_1 and S_2, the former directed perpendicular to the axis KL, and the latter along this axis. The force S_2 will give no effect since it is destroyed by the resistance of the fixed point O. The effect on the action of the force S_1, however, is given by the rule of precession, which we already know. Under the action of this force, the rotor axis KL will undergo precession in the horizontal plane. In order to determine the direction of the precession, it is necessary to rotate the direction of the force S_1 through 90° about the axis KL in the direction of rotation of the rotor.

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In Fig.13 it is assumed that the rotor is represented by the counterclockwise arrow if it is viewed from the end L of the axis KL; in the same sense, we rotate the line of action of the force S_1 through $90°$ about the axis KL. This determines the direction of the horizontal displacement of the point A at which the force S is applied. The magnitude of the angle of rotation of the axis KL during the time of action of the force S is determined by the formula

$$\mathcal{L} = \frac{S_1 a \, \Upsilon}{J \, w}$$

where $a = CA$, and the symbols J, w, and Υ have their previous meanings.
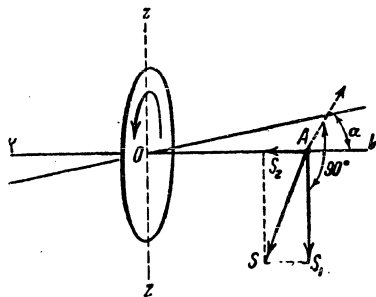
Fig.13

Section 7. Stability of a Rapidly Rotating Astatic Gyro with Three Degrees

of Freedom

Having established the law of precession, we pass now to the explanation of the stability or rigidity imparted to a gyro with three degrees of freedom by the rapid rotation of its rotor.

Let us take a model of the astatic gyroscope in a Cardanic suspension with three degrees of freedom, place the gyro axis horizontally, and put the rotor into rapid proper motion about this axis. Then let us strike the outer ring at the point A, directing the tap vertically downward. In this case, the impact will cause the vertical force S to be applied to the gyro axis at the point A (Fig.14) and to act during the course of a negligibly small time interval Υ (a negligible fraction of a

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second).

The effect of the action of this force is known to us. During the time of the impact, the gyro axis will be rotated in the horizontal plane through the angle . We determine rotation by rotating the direction of the force S through $90°$ about the gyro axis in the sense of the proper rotation of its rotor; it is assumed in Fig.14 that the proper rotation of the rotor is clockwise if viewed from the end A of the gyro axis; in this case, the rotation of the gyro axis in the horizontal plane through the angle appears clockwise if the instrument is viewed from above. The value of the angle α is determined by the formula
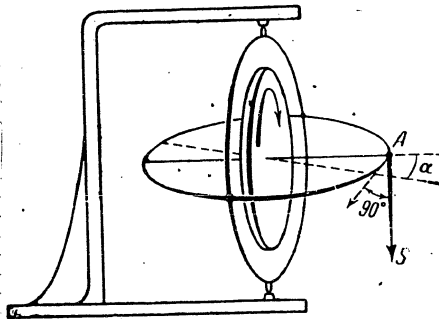
$$\alpha = \frac{Sa\gamma}{Jw}$$

where J is the moment of inertia of the rotor, w the angular velocity of the proper rotation of the gyro, and a the distance of the point A from the point of intersection of the Cardanic axes, i.e., the radius of the inner ring.

Fig.14

After the time γ of the impact has elapsed, the action of the force S is interrupted. We must conclude that the rotation of the rotor axis is interrupted simultaneously. Indeed, the considerations in Sections 5 and 6 clearly show that the rotation of the rotor axis assumes the existence of a force or a couple, applied to this axis. Consequently, if there is no such force, there can also be no such rotation of the rotor axis. This means that when the time of impact γ has ended, the rotation of the gyro axis will be instantaneously interrupted with the disappearance of the force S, and the gyro axis will remain in the new position in which it is lo-

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cuted at the instant of the termination of the impact. As for the angle α, it is clear from the formula

$$\alpha = \frac{Sa\tau}{Jw}$$

that, if the time τ is negligibly small and if the angular velocity of the proper rotation of the rotor is high, the value of this angle will be very small. The more rapidly the gyro rotor rotates, i.e., the greater the angle w, the greater will be the denominator in this formula and, consequently, the smaller will be the angle α. In the case of a very rapid rotation of the gyro rotor, the angle α proves to be so small that the rotation of the gyro axis through this angle, on impact, escapes detection by the observer. In that case, the gyro will appear to be absolutely rigid and in no way responding to the action of the impact. This is how we explain the stability or rigidity of a gyro with three degrees of freedom, whose rotor has been placed in rapid rotation.

We recommend that the reader who has available a model of a gyro in Cardanic suspension repeat the experiment we have just described and center his attention on the insignificant rotation of the gyro axis in the plane perpendicular to the direction of the impact.

We pass now to the case when the impact is inflicted not on the inner ring but on the outer ring of the Cardanic suspension. Let us strike the outer ring at the point B in a horizontal direction, with a tap from the left. At the point B, the horizontal force S, directed leftward, will be applied and will act during the course of the negligibly small time of impact τ (Fig.15). Through the intermediate stage of the inner ring, the action of the force S is transmitted to the gyro axis AC; this force may be considered to have been applied to the gyro axis at the point A, directed perpendicularly to the axis AC in a horizontal plane, as shown in (Fig.15).

Then, applying the rule of precession, we rotate the direction of the force S

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applied to the gyroscope axis at the point A through an angle of 90° about this axis in the sense of rotation of the gyro rotor (in Fig.15, it is assumed that the rotor is rotating clockwise if viewed from the A end of the gyro axis). As will be seen
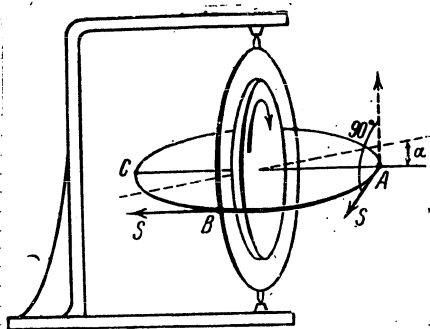
Fig.15

from Fig.15, during the time γ, there is a rotation of the gyro axis in the vertical plane through the angle α; the end A of this axis is somewhat raised and the end C somewhat lowered. The value of the angle α is determined by the previous formula

$$\alpha = \frac{\xi a \, \gamma}{J \, w}$$

where the symbols J, w, and a have their former meanings.

After the lapse of the impact time γ', the gyro axis will remain in its new position.

We advise the reader who has a model of a gyro in Cardanic suspension at his disposition to perform a series of experiments, tapping the outer and inner rings of a rotating gyroscope in various directions and noting how the axis of the gyroscope reacts to these impacts. It is very instructive to observe how the rule of precession is obeyed in all these cases.

Section 8. Nutational Oscillations of the Gyro Axis

In performing experiments with the model of a gyro with three degrees of freedom, it may be noted that, in addition to the rotation of the gyro axis in a plane perpendicular to the direction of the impact, which has already been mentioned in the preceding Section, there are also very rapid and minute vibrations of the gyro axis,

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which appear after the impact and which gradually disappear thereafter. These vibra-
tions of the gyro axis are termed nutational oscillations and have already been men-
tioned in Section 2.

We have said nothing of the nutational oscillations in the preceding Section.
They remained outside of our field of view in the theoretical picture of the gyro-
scope behavior that we draw in Section 7. This was because these oscillations are
likewise due to a manifestation of the inertia of those parts of the gyro that were
taken into consideration in the theoretical discussion given in Sections 5 and 6.
From these considerations we deduced, in Section 7, that, at the end of the impact
time τ , the motion of the gyro axis would instantaneously cease on disappearance of
the force acting during the time of impact. Such instantaneous cessation of motion
would be contrary to the law of inertia and, in actuality, could not take place.
The result of the inertia of the gyro parts is that, at the moment of completion of
the time of impact τ , the gyro axis does not stop instantaneously, but begins in-
stead to execute minute nutational oscillations which only gradually die out under
the influence of friction on the axes of the gimbal rings.

It follows from this that the analysis given in Sections 5 and 6 is not absolute-
ly exact. We did not take into account certain manifestations, secondary in impor-
tance, of the inertia of the gyro parts. It must be borne in mind that the inaccura-
cy thus admitted is smaller, the more rapidly the gyro rotor rotates; for a very rap-
idly rotating gyro, our analysis and the conclusions drawn from it may be considered
rather accurate.

In particular, the more rapidly the gyro rotor rotates, the smaller will be the
nutational oscillations of the gyro axis. For the rapidly rotating gyro with which
we have to do in the technical applications of the gyroscope, the nutational oscil-
lations are vanishingly small and have no practical significance whatever. In all
of the following discussions, these oscillations will not be further considered.

In Section 2, it was mentioned that a gyro with three degrees of freedom looses

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its stability in the position when the gyro axis coincides with the axis of rotation of the outer ring. This fact as well represents a manifestation of the inertia of the rings of the Cardanic suspension, which has not been taken into account.

Section 9.   Instability of a Gyro with Two Degrees of Freedom

Let us now turn to an explanation of the cause for the fact discovered by us that a gyro with two degrees of freedom is unstable, no matter how rapidly its rotor is rotating.

Let us analyze the experiment described in Section 4. Let us place the gyroscope axes AB (Fig.16) horizontally, put the rotor into rapid rotation, and then, holding the outer ring at the points C and D, let us strike the inner ring a vertical blow at the point A. During the period of the negligibly short time of impact, the vertical force S will act at the point A. We assume it to be directed vertically downward.

We already know what the result of the action of the force S would be if the outer ring could rotate freely about its vertical axis. The gyro axis AB would rotate, during the time of impact, through a small angle in the horizontal plane (cf. Fig.12). In this case, the outer ring would rotate through the same angle about the vertical axis MN; the points C and D of this ring would be somewhat displaced in the horizontal plane (the point C leftward, the point D rightward, if the gyro rotor were rotating as assumed in Fig.16 and Fig.12). However, the rotation of the outer ring is prevented by the hand of the experimenter, who firmly holds the outer ring at the points C and D. In reality, the outer ring cannot rotate, but exerts on the hands of the experimenter holding this ring, pressures acting on the points C and D, as has just been stated.

However, action and reaction are equal. If the hand of the experimenter is subjected to forces on the side of the outer ring held by this hand, then, at the same time, the forces F applied by the experimenter's hand will act at the points C and D

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of the inner ring; these forces are the same in magnitude but opposite in direction (Fig.16). The forces F, applied to the outer ring, are transmitted by means of the inner ring to the gyro axis. Thus, the horizontal forces F applied to the axis of the gyroscope arise at the points A and B.

Let us now apply the law of precession of these forces. By rotating the directions of the force F applied to the points A and B, through an angle of 90° about the
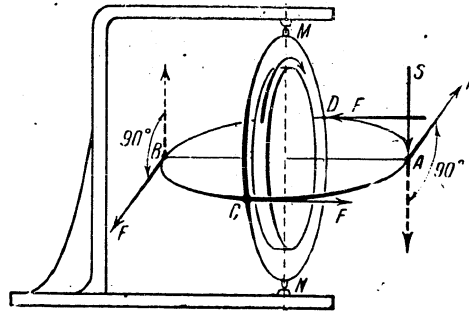
Fig.16

axis AB in the same sense as the rotation of the gyro rotor, the action of the forces F will cause the gyro axis to acquire a rotation in a vertical plane, so that its end A is lowered and its end B is raised.

Thus, as a result of the action of the horizontal forces F, a rotation of the entire instrument about the horizontal axes CD is produced. As a result of the inertia effects mentioned in Section 8, this rotation cannot cease instantly on cessation of the time impact or on interruption of the force of impact S. At the same time, fixing the outer ring at the points C and D causes a prolonged action of the horizontal force F which, on transmission at the points A and B to the gyro axis ensure its further prolonged rotation in the vertical plane about the axis CD.

Thus, the result of the vertical impact applied to the outer ring at the point A

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is a prolonged rotation of the instrument about the horizontal axis CD (which is then gradually damped under the influence of the friction on the axis CD). In this case, the rapidly rotating gyro behaves exactly as though its rotor had been given no proper rotation whatever. The rapidly rotating gyro with two degrees of freedom is entirely deprived of the power of resisting the action of forces tending to change the direction of its axis. As is clear, the instability of a gyro with two degrees of freedom is caused by the horizontal forces F produced by fixing the outer ring.

Section 10. Precession of a Gyro Due to a Continuously Acting Force Applied to its

<p align="center">Axis</p>

Until now, we have experimented with a model of the free or astatic gyroscope to which no other external forces were applied except impact forces, acting for a negligibly short interval of time. Let us now pass to the study of the behavior of a gyro with three degrees of freedom to whose axis a continuously acting external force is applied.

Using our model gyro in Cardanic suspension, the gyro axis AB is placed horizontally, and a rapid rotation about the axis AB is imparted to its rotor (Fig.17). Of course, the axis AB remains motionless. We now carefully suspend a small weight from the inner ring at the point A. The axis of the gyroscope AB immediately begins to "deviate", precessing in a horizontal plane; the entire instrument instantly starts rotating about the vertical line zz.

It is now no longer difficult to explain this phenomenon. The point A on the gyro axis is constantly subjected to the action of the force P directed vertically downward, and equal to the weight of the counterpoise. Let us apply the rule of precession. By rotating the force P about the axis AB through 90° in the sense of rotation of the gyro rotor (in Fig.17, it is assumed that the rotor is rotating clockwise if viewed from the A end of the gyro axis). We see that, according to the rule of precession, the axis begins to rotate in a horizontal plane (with its rota-

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tion about the line zz being clockwise, if viewed from above). During the negligibly small time interval τ, the axis AB rotates through the small angle α determined by eq. (3) (cf. Section 6):

$$\alpha = \frac{Pa\tau}{J w}$$

where J is the moment of inertia of the rotor, w the angular velocity of its proper motion, and a the distance of the Point A from the point of intersection of the Cardanic axis (i.e., the radius of the inner ring).
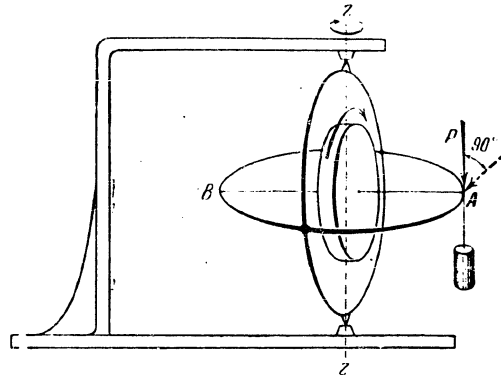
Fig.17

In the following and in all subsequent time intervals τ, the same processes are repeated. As a result of the continuous action of the force P, a continuous rotation of the entire instrument about the line zz is established.

Such a rotation of the gyro about the line zz is termed its precession, and the line zz is called the axis of precession. The direction of the precessional rotation about the axis is shown in Fig.17 by the curved arrow placed near the zz axis. Let us denote the angular velocity of the precessional rotation about the zz axis by the symbol w_1. It is equal to the ratio of the angle of rotation α to the corre-

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sponding time τ ; bearing in mind eq. (3), we obtain the result that

$$w_1 = \frac{\alpha'}{\tau'} = \frac{P\,a}{J\,w} \qquad (4)$$

This formula determines the angular velocity of precession w_1.

It follows from eq. (4) that the higher the angular velocity of the proper rotation of the rotor w, the smaller will be the angular velocity of precession w_1. As a result of the unavoidable friction in the instrument, the angular velocity of proper rotation w gradually decreases with the passage of time. Consequently, the angular velocity of precession w_1, must gradually increase in time. We very strongly advise the reader who has a gyro model available to reproduce the experiment here described and to convince himself of the gradual increase in angular velocity of precession as the rate of proper rotation of the gyroscope decreases.

As for the sense in which precession occurs, the reader will easily find by repeating our reasoning, that a change of the direction of proper rotation of the rotor to the opposite sense, will result in a precession which will likewise be in the opposite sense: If, in Fig.17, the rotor is represented as rotating counterclockwise about the axis AB (if viewed from the A end of this axis), then the precessional rotation about the axis would likewise be counterclockwise (if viewed from above). It is instructive to perform this experiment a number of times, imparting to the rotor a proper rotation in different senses and observing the variation in the sense of precession, as a function of the sense of proper motion.

In reproducing this experiment, the attentive observer will note that the precessions of the gyro are accompanied by small and very rapid oscillations of the gyro axis. These are the nutational oscillations, with which we are already acquainted and which have been mentioned above in Section 8. The more rapidly the gyro rotor rotates about its axis, the smaller will be these nutational oscillations of the gyro axis.

In school physics laboratories, gyro models made of a bicycle wheel with a built-

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up axle are frequently used. Let us impart to the wheel a sufficiently rapid rota-
tion and suspend the instrument from the cord AC, attached to some point A of the
AE axis of the gyro, giving its axis a horizontal direction (Fig.18). It cannot but
surprise us that the gyro fails to descend under the action of the force of gravity
P. Instead, its axis AB begins to precess in a horizontal plane, rotating about the
vertical line AC.
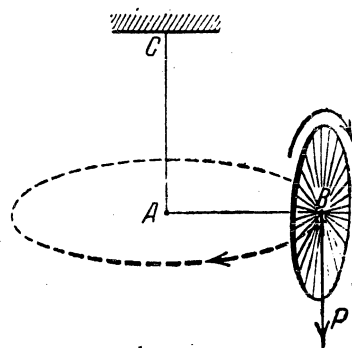
This is the well-known phenom-
enon of precession which, in this
particular case, is due to the ac-
tion of the force of gravity P on
the gyroscope, which here plays
the same role as the suspended
counterpoise in the preceding ex-
periment. The sense of the preces-
sion is determined, as always, by
the rule of precession (in Fig.18

Fig.18

the sense of the proper rotation

of the gyro and the sense of precession are shown by arrows). The angular velocity
of precession w_1 is found from eq. (4):

$$w_1 = \frac{Pa}{Jw}$$

where a = AB; J is the moment of inertia of the wheel; and w is the angular veloc-
ity of its proper rotation about AB axis.

We remark again that, according to this formula, the angular velocity of preces-
sion w_1 is smaller, the larger the proper angular velocity w becomes, and vice
versa. It is very instructive, in performing the experiment with the bicycle wheel,
for the observer to note how the precessional rotation about the vertical AC gradu-
ally increases as the angular velocity of the proper rotation of the wheel decreases.

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If a gyro model made out of a bicycle wheel is unavailable, the above experiment may be reproduced with a toy model of the gyro, consisting of a top M whose axis is fixed to the ring N (Fig.19). It is easy to find such toy gyros in shops.

Section 11. Gyro with Three Degrees of Freedom on a Rotating Base. The Foucault Gyro. Experimental Proof of the Earth's Rotation.

We know that a rapidly rotating astatic gyro with three degrees of freedom possesses a high degree of stability; its rapid rotation gives it the power of energetically resisting the action of impacts tending to vary the direction of its axis. This same property is possessed by a gyro with three degrees of freedom when placed on some rotating base.
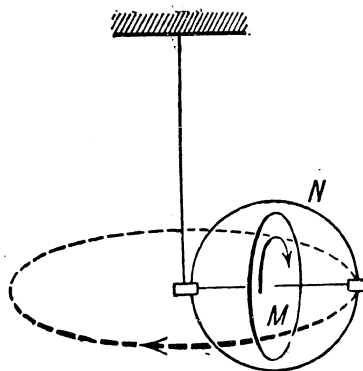
Fig.19

Let us place our model of the free gyro with three degrees of freedom on the small platform L, which may be rotated about the vertical axis MN (Fig.20). Let us impart to the gyro rotor a rapid rotation about the axis AB and note the direction of this axis. We then begin to turn the stage L of the instrument. The rotation of the stage is in no way reflected in the direction of the axis AB. The axis AB is not involved in the rotation of the base of the instrument; it stably preserves an invariant direction in space*.

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* For this experiment, a so-called centrifugal machine, available in most school physics laboratories, may be used. If this is not available, the base of the gyroscope may be simply rotated by hand.

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Thus, the axis of a rapidly rotating astatic gyro with three degrees of freedom, placed on a rotating base, stably preserves its constant direction in space. A number of technical applications of the gyro, discussed in the following Chapter, are based on this property.

It must be borne in mind that, in essence, all gyro with which we have to do are placed on a rotating base since their common base is the rotating earth. The following conclusions must be drawn from the above statements. If it were possible to construct a gyro of such ideal perfection as to be completely free of the action of forces causing precession (i.e., a gyro ideally balanced and completely free from friction in the Cardanic axes), then the axis of such an ideal gyro, when in rapid rotation, would stably maintain a constant direction in space.
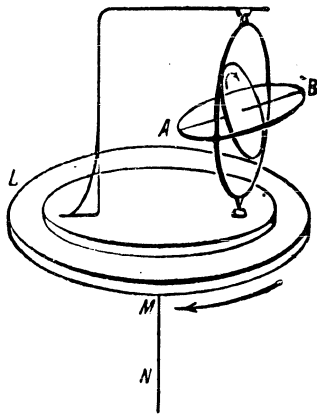
If we directed the axis of such a gyro toward any fixed star, and if the instrument were operating a long time, the axis of the gyro would follow the apparent diurnal motion of the star in the celestial vault (which is the consequence and the manifestation of the diurnal rotation of the earth). By pointing the gyro axis toward the eastern part of the celestial vault (where the stars rise), we would see that the gyro axis would slowly rotate from east to west and would simultaneously rise above the horizon; the gyro axis directed toward the western part of the celestial vault (where the stars set) would rotate from east to west, and would gradually sink toward the horizon.

Fig.20

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In this ideal case, the gyro axis would thus be continuously displaced with respect to terrestrial objects. Such an apparent displacement of the gyro axis, reflecting the actual diurnal rotation of the earth, would be a illustrative proof of the rotation of the earth. This was the thought on which Foucault based his famous experiment reported by him to the Paris Academy of Sciences in 1852.

The difficulty of the experiment consisted in the construction of a gyroscope approximating the ideal perfection described above. In the Foucault gyro, the outer gimbal ring was suspended on a thin untwisted fiber strand (Fig.21) and rested on a guide bearing; the inner ring lay inside the outer ring, resting on two bearings designed in the form of two knife blades. All measures were taken to make the instrument completely balanced, i.e., to have the center of gravity of the instrument coincide with the point of the intersection of the Cardanic axes. The instrument parts were so labile that (as Foucault said in his report on this experiment) they would be set in motion by the slightest breath.
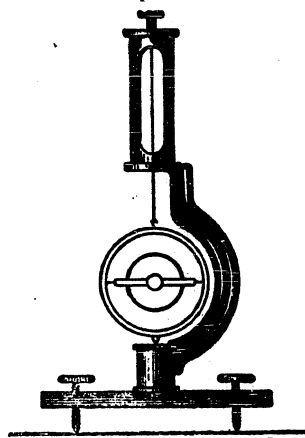
Fig.21

The gyro rotor was placed in rapid rotation. After the instrument had come to rest, observations were made on the subsequent behavior of the outer ring. We already know that, in the absence of any forces acting on the instrument, its outer ring, together with the gyro axis, must slowly rotate from east to west, and that this apparent rotation of the gyroscope reflects the true diurnal rotation (from west to east) of the earth.

Foucault actually did succeed in detecting, on his instrument, this slow rota-

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tion of the outer ring. In this way, for the first time, the diurnal rotation of the earth was demonstrated by means of a pure laboratory experiment.

Section 12. Gyro with Two Degrees of Freedom on a Rotating Base. Foucault's Rule.

We have seen that a rapidly rotating gyro with three degrees of freedom on a rotating base, stably preserves the constant direction of its axis in space. This property is completely lacking in a gyro with two degrees of freedom. Let us consider the behavior of such a gyro when placed on a rotating base.
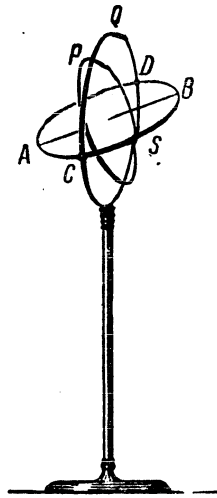
Many physics laboratories of schools use a model of the gyroscope with two degrees of freedom. This consists of the top (rotor) P, whose axis AB is attached to the ring S whose horizontal axis of rotation CD, in turn, is attached to a fixed vertical ring Q which is rigidly connected to the base of the instrument (Fig.22). This represents a gyro in a Cardanic suspension, whose outer ring Q is made immobile. It is clear that such a gyro

Fig.22

has two degrees of freedom.

Let us place this gyro on the stage L which may be put in rotation about the vertical axis MN (Fig.23). Let us give the gyro axis AB a horizontal direction and place the rotor in rapid proper motion about the axis AB. Then, let us begin to turn the turntable L. The axis of the gyro will leave its horizontal position and slowly assume a vertical position. Let us now change the sense of rotation of the turntable L. The gyroscope immediately will rotate through 180° about the axis CD,

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and its own axis will return to a vertical direction, except that the end of the axis AB, previously pointing upward now points downward. Such a somersault of the gyro about the axis CD will be repeated each time the sense of rotation of the turntable L is changed.

Let us analyze this phenomenon.

We place the gyro axis AB in a horizontal position and impart to the gyro rotor the proper rotation and the sense shown in Fig.23 by the arrow (which is clockwise if viewed from the B end of the axis AB). The stage L is then rotated clockwise if viewed from above (as shown in Fig.23); at the same time the ring S is fixed at the points A and B, without permitting it to leave this horizontal position. The problem is to define the force to be applied at the points A and B. The axis of the gyroscope will now rotate in the horizontal plane about the vertical axis MN. As we already know (cf. Section 5), in order to effect this motion it is necessary to apply a couple of vertical forces to the gyro axis. Recalling Fig.11 in Section 5, we see that, at the point B (Fig.23), the vertical force F directed downward, and at the point A the equivalent force, but directed upward, must be applied (we would reach the same conclusion as to the direction of the forces at the points A and B by applying the rules of precession directly to the given case). Thus, in order to force the gyro axis to remain horizontal, it is necessary to press downward at the point A. This means that the axis AB tends to rise at its B end and sink at

Fig.23

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its A end. It becomes understandable from this that, if the ring S is freed, thus eliminating the forces F, then the B end of the gyro axis will actually rise and the A end will sink; the gyro axis will then become vertical, with the B end on top (Fig. 24).

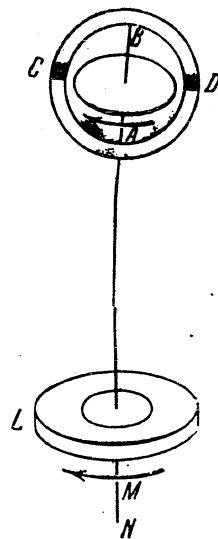We conclude that the axis of a gyro with two degrees of freedom, on a rotating base, becomes parallel to the axis of rotation of the base in such a way that the rotation of the rotor and the rotation of the base are both in one and the same sense.

Let us agree to term the two parallel axes of rotation simultaneously parallel if the rotation about these axes is directed in one and the same sense. On introducing this term, we arrive at the following formulation of the result obtained: The axis of a gyroscope with two degrees of freedom, on a rotating base, tends to simultaneous parallelism with the axis of rotation of the base.

This result, which was first enunciated by Foucault, is generally known as Foucault's rule.

Fig.24

It is now easy to understand what takes place when the sense of rotation of the base is changed. In obeying the same rule of Foucault, our gyroscope would have to flip over immediately to make its axis again proportionally parallel to the axis of rotation of the base. Each change in sense of rotation of the base of the instrument will cause another inversion of the gyro.

If no special model of the gyroscope with two degrees of freedom is available, the experiment described in this Section can be reproduced by means of an ordinary

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gyro in a Cardanic suspension. After placing the rotor in rapid rotation, the outer ring must be rotated by hand about the vertical axis in alternating directions. The gyroscope will respond without lag to these rotations, by executing successive inversions in accordance with the Foucault's rule.

Section 13. Derivation of the Formula for the Gyroscopic Moment

In Section 5, we have made the reader acquainted with eq. (2) for the value of the gyroscopic moment:

$$M = Jw \ \frac{\alpha}{\tau} = Jw \ \ w_1$$

This formula determines the value of the moment of the forces that must be applied to the axis of a gyroscope to cause it to rotate through the angle α during the time τ about an axis perpendicular to it. Here, J is the moment of inertia of the gyro rotor, W the angular velocity of its proper rotation, and $w_1 = \dfrac{\alpha}{\tau}$ the angular velocity of rotation of the gyro axis.

Let us now dwell on the derivation of this formula:

First, we will revert to the beginning of Section 5. There, we discussed the effect, on the velocities of four points A, B, C, D lying on the rotor periphery, of a rotation of its axis through the small angle α about the line zz (Fig.6). Let us now discuss the same question with respect to any point lying on the rotor circumference.

Let us take, on the rotor periphery, any point N and let us denote by φ the angle formed by the radius ON and the horizontal radius OA (Fig.25). As a result of the proper rotation of the rotor about its axis KL, the point N will have the velocity v, directed along the tangent to the rotor periphery (Fig.25 and 26), and equal to $v = R \ w$, where R is the radius of the rotor and w is the angular velocity of its proper rotation. The proper rotation of the rotor will be considered as clock-

wise of viewed from the L end of the axis KL (Fig.25).

During the interval of time τ (which we assume to be infinitesimal), the gyro axis KL will rotate in the horizontal plane through the small angle   and will take the position $K_1 L_1$; at the same time, the plane of the rotor, rotating through the same angle α about the vertical line zz, will take the position $A_1 BC_1 D$ (Fig.25). Then N, during the time τ , will be displaced to the position N_1, and its velocity, without changing in magnitude, will change its direction; after the rotation, the velocity of the point N_1 (which we shall denote by the symbol v_1, where $v_1 = v$) is directed along the tangent to the circumference $A_1 BC_1 D$ (Fig.25).

For our later argument we must now ascertain the value of the angles formed by the direction of the velocity v_1 with the original direction of the velocity v. Let the point of intersection between

Fig.25

the velocities v and v_1 and the line zz be denoted by the symbol O_1 and let us drop the perpendiculars NO_1 and $N_1 O_2$ from the points N and N_1 to the line zz. Noting that, in the triangle $NO_1 O_2$, the angle at the vertex O_1 is equal (as a result of the sides being perpendicular) to the angle φ, we conclude that

$$NO_2 = NO_1 \sin \varphi \tag{5}$$

During the time τ , the point N is displaced by NN_1, which is an arc corresponding to the central angle $NO_2 N_1$. Since this angle is equal to the angle of rotation

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of the plane of the rotor, i.e., to the angle α , and since the arc is equal to the products of the radius and the central angle, then

$$NN_1 = NO_2 \qquad\qquad (6)$$

On the other hand, the same displacement of NN_1 may be considered with sufficient accuracy as the arc corresponding to the central angle in O_1N_1. This is the angle we require between the directions of the velocities v and v_1; let us denote it by the letter x. Then we shall have

$$NN_1 = NO_1 \cdot x \qquad (7)$$

From eqs. (6) and (7) we find

$$NO_1 \cdot x = NO_2 \cdot \qquad ,$$

whence

$$x = \frac{NO_2}{NO_2}$$

Fig.26

However, eq. (5) yields

$$\frac{NO_2}{NO_1} = \sin \varphi$$

Consequently,

$$x = \alpha \ \sin \varphi \qquad\qquad (8)$$

This is the value of the angle between the velocities v and v_1.

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Let us now represent the triangle NN_1O_1* on a special diagram (Fig.27) and let us construct at the point N, a parallelogram in which the segment v is one of the sides while a segment equal and parallel to v_1 is a diagonal; the second side of this parallelogram will be denoted by u_1. The transition from the velocity v of
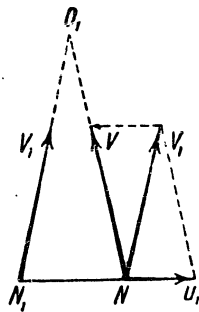
Fig.27
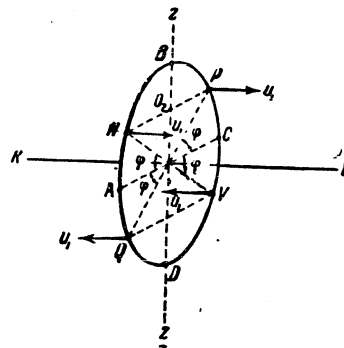
Fig.28

point N to the velocity v_1 of point N_1 is equivalent to the appearance at point N of a new velocity component u_1 which, on combining with the velocity v, yields a new velocity v_1. Since the triangles are similar, we have:

$$\frac{u_1}{NN_1} = \frac{v}{NO_1}$$

whence

$$u_1 = v \frac{NN_1}{NO_1}$$

or, on the basis of eqs. (7) and (8),

$$u_1 = vx = v\ \alpha\ \sin\ \varphi \tag{9}$$

* The arc NN_1 may be replaced, with sufficient accuracy, by a rectilinear segment.

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As for the direction of the velocity u_1, it will be seen from the diagram (Fig. 27) that it is opposite to the direction of the displacement NN_1. In view of the smallness of the angle α , this displacement may be considered perpendicular to the plane of the circumference ABCD (Fig.25), i.e., parallel to the rotor axis KL. Consequently, the velocity u_1 is directed perpendicularly to the plane of the rotor, or parallel to its axis KL (Fig.28).

We have assumed the point N to belong to the quadrant AB of the rotor. Let us now take, on the quadrant BC of the rotor, the point P, symmetric to the point N with respect to the line zz. Repeating the same reasoning, we find easily that, on rotating the plane of the rotor about the line zz through the angle α , the new velocity component u_1 will appear at the point P and will be equal in magnitude and direction to that at the point N. At the points Q and V, belonging to the semicircumference ADC and symmetric to the points N and P with respect to the line AC, there will appear the same velocities A_1, equal in magnitude but opposite in direction.

Thus, at all points of the semicircumference ABC, will appear the velocities u_1, determined by eq. (9) and directed parallel to the rotor axis KL toward the L end of this axis. The velocities u_1, equal in magnitude but opposite in direction, will appear at the points of the semicircumference ADC. The appearance of the velocities u_1 is due to the rotation of the rotor plane through the angle α , in the time τ , about the line zz. Let us now take into account the other effect, which is likewise a consequence of this rotation.

Let us return to the point N. In participating in the rotation of the rotor about its axis KL, this point has a rotational velocity about the axis KL, which we have denoted by the symbol v. However, during the time τ , the plane of the rotor is likewise rotated about the line zz through the angle α , and as a result of this, the point N, which participates in this second rotation, likewise receives a rotational velocity (which we shall denote by the symbol w) about the line zz. The

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velocity w is equal to the product of the radius NO_2 and the angular velocity of rotation of the rotor plane about the line zz; denoting this angular velocity (as in Section 5) by w_1, i.e., assuming that

$$w_1 = \frac{\alpha}{\tau}$$

we have (Fig.28)

$$w = NO_2 \qquad w_1 = NO_2 \frac{\alpha}{\tau}$$

Fig.29

However, the direction of the velocity w is perpendicular to the plane of the rotor, i.e., parallel to the axis KL; in addition, at the sense of rotation of the rotor plane, adopted by us, about the line zz, the velocity w is directed toward the K end of the axis KL, i.e., is opposite in direction to the velocity u_1 at the point N.

It must be taken into account that, during the time τ, the point N while participating in the rotation of the rotor about the axis KL, traverses the short path $NN' = v\tau$, and toward the end of the time interval will be at the distance $N'O'_2$ from the line zz (Fig.29 and Fig.30) instead of at the distance NO_2. In other words, during the time τ, the radius NO_2 is diminished by the quantity Nn (Fig.30) and the velocity v by the value Nn α/τ. This is equivalent to the appearance of a new velocity component $u_2 = Nn \; \alpha/\tau$, opposite in direction to the velocity v, i.e., coinciding in direction with the veloc-
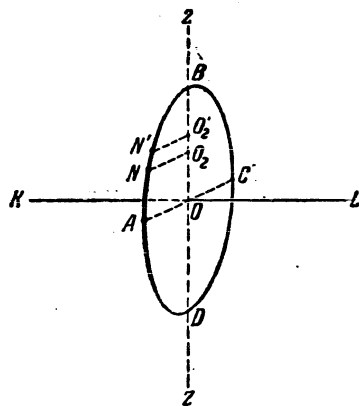
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ity u_1. We note now that, in the triangle $NN'n$ (Fig.30), the angle at the vertex N' is equal (since the sides are perpendicular) to the angle φ and that, consequently,

$$Nn = NN' \sin \varphi = v \, \tau \sin \varphi$$
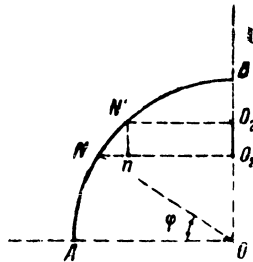
whence

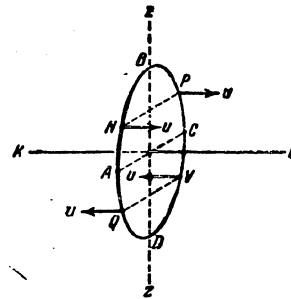$$u_2 = Nn \frac{\alpha}{\tau} = v \, \alpha \sin \varphi$$

Fig.30   Fig.31

We see that the velocities u_1 and u_2 coincide not only in direction but also in magnitude. On compounding these two velocities, we conclude that, on the rotation of the rotor plane about the line zz through the angle α in the time τ, the following velocity component appears at the point N during this time:

$$u = u_1 + u_2 = 2v \, \alpha \, \sin \varphi \tag{10}$$

which is directed parallel to the axis KL toward the end L (Fig.31).

In exactly the same way, we may convince ourselves that, owing to the effect now under consideration, the velocities u_1 acquired by the points P, Q, and V are now

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doubled (Fig.28). At all points of the semicircumference ABC there appear the velocities u parallel to the KL axis and directed toward the end L; the same velocities, but in opposite directions, appear at the points of the semicircumference ADC (Fig.31).
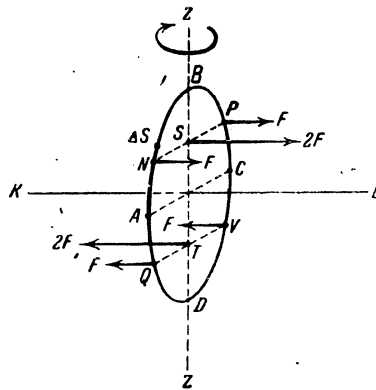
Let us now revert to Newton's Second Law of Motion. Let us assume, as in Section 5, that the entire mass of the rotor is concentrated along its periphery ABCD, on which it is uniformly distributed. Let m be the mass of the rotor, R its radius, and let us divide the entire length of the circumference $2 \pi R$ into infinitesimal elements Δ s (Fig.32). To each element of arc Δ s corresponds the elementary mass ,

Fig.32

which is found from the proportion

$$\frac{\mu}{m} = \frac{\Delta s}{2 \pi R}$$

whence

$$\mu = \frac{m \Delta s}{2 \pi R}$$

On rotation of the rotor plane about the lines zz through the angle α during the time τ , the elementary mass μ at the point N has its velocity changed by the value $u = 2v_\alpha \sin \varphi$, directed parallel to the axis KL toward the end L. consequently, in accordance with Newton's Second Law, this elementary mass, during the time τ , is subjected to the force F which is likewise directed parallel to the

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rotor axis toward the end L, and which is equal to

$$F = \mu \frac{u}{\tau} = \frac{m}{\tau} \frac{\Delta s}{R} v \frac{\alpha}{\tau} \sin \varphi$$

Denoting, as above, the angular velocity of the proper rotation of the rotor about its axis KL by w , and the angular velocity of rotation of the rotor plane about the line zz by w_1, we have

$$v = R \ w \ , \ \frac{\alpha}{\tau} = \ w_1$$

and, consequently,

$$F = \frac{1}{\pi} m \ w \ w_1 \ \Delta s \sin \varphi \qquad (11)$$

The same force F is applied at the point P, and forces equal to F but opposite in direction are applied at the points Q and V (Fig.32). On compounding the two forces F applied to the points N and P, we obtain their resultant 2F, applied at the point S, which bisects the segment NP; in exactly the same way the compounding of the forces F, applied at the points Q and V, yields the force 2F in the opposite direction applied at the point T bisecting the segment QV. The two forces at the points S and T form a couple of forces with the arm ST; the moment of this couple is equal to 2F · ST.

Let us substitute here the value of F taken from eq. (11). We note, in this case, that the quantity $\Delta s \sin \varphi$ has a simple value. On laying off (Fig.33) from the point N the element of arc NN' $= \ \Delta s$ and producing horizontal and vertical lines through the point N and N', we obtain the elementary triangle NN'n, in which the angle at the vertex N' is equal to φ ; from this triangle we find

$$\Delta s \sin \varphi \ = Nn$$

Thus, we have

$$2F \cdot ST = \frac{2}{\pi} m \ w \ w_1 \ ST \cdot Nn$$

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However, the product ST · Nn is the area of the zone NnqQ (which is hatched in Fig.33); denoting this area by the symbol σ , we have

$$2F \cdot ST = \frac{2}{\pi} m \ w \ w_1$$

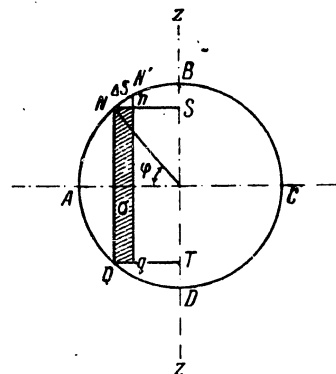This is the moment of the couple 2F, applied at the points S and T. We will have an infinite number of such couples, corresponding to all the elements of arc AB. All these couples of forces are compounded into a single resultant couple in the vertical plane, passing through the axis KL and the line zz, whose moment is equal to the sum of the moments of the component couples. Consequently, denoting the moment of the resultant couple by the symbol M and using the sign Σ to denote summation, we will have:

Fig.33

$$M = \Sigma \ 2F \cdot ST = \Sigma \ \frac{2}{\pi} m \ w \ w_1 \sigma$$

In all the components of the latter sum, there enters the common factor $\frac{2}{\pi} m \ w \ w_1$; on removing this common factor from the summation sign, we have

$$M = \frac{2}{\pi} m \ w \ w_1 \Sigma \ \sigma$$

The sum of the area of the zones σ , corresponding to all possible positions of the point N on the arc AB, gives the area of the semicircle BAD and, consequently,

$$\Sigma \sigma = \frac{1}{2} \gamma R^2$$

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Thus, we finally obtain

$$M = mR^2 \; w \; w_1$$

The quantity mR^2 is called the moment of inertia of the rotor, under the assumption that all of its mass is concentrated along its periphery. Denoting the moment of inertia by the symbol J, i.e., setting

$$J = MR^2 \tag{12}$$

we find

$$M = J \; w \; w_1 \tag{13}$$

This must be the moment of the couple of forces to be applied to the gyroscope in the vertical plane passing through its axis, in order to cause a rotation of the axis in the horizontal plane through the angle α during the time τ ; we must not forget that here $w_1 = \dfrac{\alpha}{\tau}$. This moment is, therefore, termed the gyroscopic moment.

In deriving eq. (13) which coincides with eq.(2) given in Section 5, we assumed that all the entire rotor mass is concentrated along its periphery, i.e., that the rotor has the form of a solid ring with relatively light spokes. We have done this in order to simplify the derivation. Equation (13) remains true, even at a different distribution of mass in the rotor, i.e., at a different form of the rotor. However, the value of the moment of inertia depends on the shape of the rotor. For example, if the rotor has the shape of a round disk of uniform thickness, of mass m and radius R, then its moment of inertia is determined not by eq. (12) but by the formula:

$$J = \frac{1}{2} mR^2$$

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CHAPTER II

SOME OF THE SIMPLEST APPLICATIONS OF THE GYROSCOPE

Section 14.   The Obry Gyroscopic Steering Device*

Let us now turn to the applications of the gyroscope. We will begin with a few applications of the astatic gyro with three degrees of freedom. We already know that, if such a gyro is set in rapid proper rotation, then its axis becomes able to resist energetically all forces tending to vary its direction in space. The rapidly rotating astatic gyro is irreplacable in all cases in which an instrument, stably maintaining its assigned direction in space, is required. Some of these applications of the astatic gyro will be considered by us in the present Chapter.

The widespread application of the gyro in various fields of technology commenced on the dividing line between the past and present Centuries. One of the first and very successful steps in this direction was the invention in 1898, by the Austrian engineer Obry, of a gyroscopic direction-keeping mechanism for self-propelled torpedoes. This apparatus serves to keep a torpedo moving through water on its set course.

The torpedo has a cigar-shaped form and is driven by propellers mounted in its tail. The nose of the torpedo carries the explosive charge. The body of the torpedo contains a pneumatic drive which, during the time when the torpedo is running,

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* Translator's note: In U.S. terminology, this a direction-keeping mechanism (for torpedoes).

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maintains the rotation of both propellers (rotating in opposite directions) and also contains the chamber for the compressed air which feeds the pneumatic drive. To maintain the torpedo moving through the water at the assigned depth and on the set course, two rudders are used, the depth rudder in the horizontal plane, and the directional rudder in the vertical plane. Both rudders are automatically actuated by
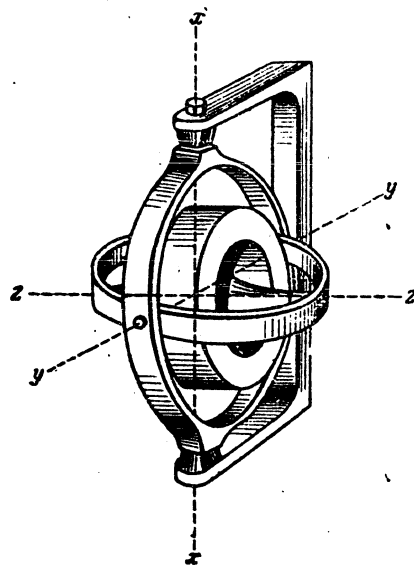
by the corresponding devices, the depth instrument and the directional instrument. Disregarding here the depth device (whose action is based on the laws of hydrostatics), we will give a detailed description of the directional device which utilizes the properties, already known to us, of a rapidly rotating astatic gyro with three degrees of freedom.

The steering apparatus of the torpedo, which controls the directional rudder, consists of a rapidly rotating astatic gyro in Cardanic suspension with three degrees of freedom (Fig.34). In the normal position of the torpedo, and the axis of rotation of the outer ring xx is vertical, the axis of rotation of the inner ring yy and the rotor of the gyroscope zz are horizontal; the axis of rotation of the rotor zz runs parallel to the longitudinal axis of the torpedo (in the direction of its motion), while the axis of rotation of the inner ring yy is

Fig.34

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normal to the longitudinal axis of the torpedo.

If the torpedo is moving correctly toward its target along the set course, then
the gyro axis zz, preserving its direction in space, will always be directed along
the longitudinal axis of the torpedo (Fig.35)*; the plane of the outer ring of the
suspension has a direction normal to the longitudinal axis of the torpedo. The sit-
uation is different if for any reason at all, the torpedo deviates from the set
course. Let us assume that the torpedo deflects from the direction to the target by
the angle α to the left (Fig.36). The gyro axis zz, which stably maintains its
direction in space, will as before point toward the target and will form the angle
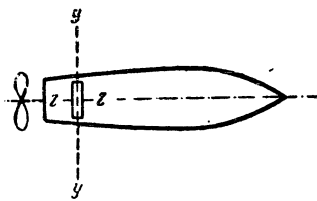
Fig.35
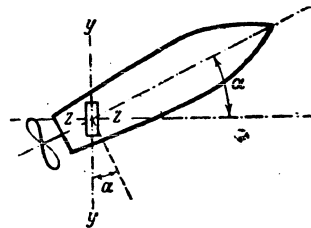
Fig.36

α with the longitudinal axis of the torpedo. With respect to the body of the tor-
pedo, the gyroscope then executes a deviation about the vertical axis xx by the an-
gle α to the right.

The body of the torpedo contains a servomotor which is actuated by compressed
air from the air chamber and triggers the directional rudder. Feeding of the com-
pressed air to the servomotor is regulated by a valve connected with the gyro (more
specifically, with the outer ring of the gyro suspension). When the valve is in its
central position, the air supply to the servomotor is interrupted, and the rudder is

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* Figures 35 and 36 give a view of the torpedo from above. The position of the
gyro rotor is shown schematically; the gimbals are not shown at all.

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motionless. As soon as the gyroscope is displaced with respect to the body of the torpedo and turns, let us say, to the right, it will produce a displacement of the valve and open the air intake of the servomotor. This deflects the rudder, also producing a rotation of the moving torpedo to the right. At an opposite rotation of the gyro from its central position, in exactly the same way, a rotation of the rudder in the opposite direction is produced.

Thus, any deviation of the torpedo from the set course will automatically actuate the directional rudder, causing the torpedo to turn in the opposite direction and thereby correcting the deviation of the torpedo from the set course. However, this is not all: Any deviation from the course to the left is followed by a deviation of the course to the right, accompanied by a new deviation to the left, and so on. The path of the moving torpedo is not a strictly straight line but has instead the shape of a slightly sinuous line; this produces the phenomenon known as yawing of the torpedo.

In the mechanism described here, the source of energy necessary for turning the directional rudder is compressed air from the air chamber. When supplied to the servomotor the compressed air actuates it and causes deflection of the rudder. The function of the gyroscopic direction-keeping mechanism is to sense any deviation of the torpedo from the set course and automatically tripping the servomotor at the proper moment. This device automatically gives control of the correct course of the torpedo according to the principle of remote control; here the two main functions of an automatic control are realized: 1) to sense any deviation of the given object from its proper course; and 2) to supply the necessary energy to actuate those parts of the machine (the directional rudder) which will correct the existing deviation; these functions are exerted by two different instruments, the gyroscopic direction-keeping mechanism and the servomotor.

A design is likewise conceivable in which these two functions are combined in a single unit. This corresponds to the principle of direct control. Here one and the

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same instrument senses the deviations of the object from the correct course and si-

multaneously supplies the required energy to correct these deviations.

After invention of the self-propelled torpedo, attempts in this direction were immediately made. One of these is represented by the gyroscopic steering device of the Howell torpedo. In this apparatus, the rotor of the gyroscope was suspended within the torpedo, but not in a Cardanic suspension; instead, the axis of the rotor was placed in bearings rigidly attached to the body of the torpedo, so that the gyroscope had only one degree of freedom with respect to the body of the torpedo, corresponding to its own proper rotation; the axis of rotation of the gyroscope
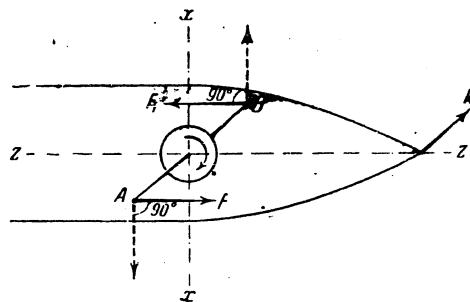
Fig.37

was directed, not along the longitudinal axis of the torpedo (as in the Obry device), but along the transverse horizontal axis (Fig.37). If it is borne in mind that the body of the torpedo itself can rotate freely about a vertical axis (corresponding to the pitching of the torpedo on its course), and about the transverse horizontal axis (rolling of the torpedo), and if these two degrees of freedom are added to the degree of freedom corresponding to the proper rotation of the gyroscope, then it must be concluded that the Howell torpedo control unit constitutes a gyroscope possessing three degrees of freedom. In order to understand the principle of operation of this gyroscope, let us imagine that any horizontal force H (e.g., the impact of

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a wave) tends to cause the torpedo to deviate from its set course (Fig.37). This force H, tending to turn the body of the torpedo about the vertical axis generates, at the bearing A and B of the torpedo axis, the horizontal forces F and F_1 acting on the ends of the gyro axis AB and tending to rotate this axis about the vertical axis xx. We already know how to find the resultant of the forces F and F_1 applied to the gyro axis; we recall the rule of precession in Section 6. According to this rule, the direction of displacement of points A and B is found by rotating the direction of the forces F and F_1 through 90° about the axis AB in the sense of rotation of the gyro rotor (in Fig.37, we assume that the rotor rotates clockwise). We conclude that, under the action of the forces F and F_1, the A end of the gyro axis is lowered while the B end is raised; at the same time, the entire body of the torpedo is rotated about its longitudinal axis zz (Fig.37). In this way the Howell torpedo, on receiving an impact H from a wave, is not deflected from its course but only acquires a certain roll and swings about its longitudinal axis zz. Later designs of the Howell control instrument included parts designed to eliminate any possible rolling of the torpedo.

Without going into further detail, we note that the idea of the direct-acting gyroscopic direction-keeping mechanism on which the design of the Howell torpedo was based did not prove itself in practical use; this design gave way to the remote-control Obry mechanism. In this latter design, the function of the gyroscope is merely to indicate the deviation of the torpedo from the set course, while the actual work of correcting the deviations is performed by a different source of energy. It is only natural that, in such a design, the gyroscope should be subject to milder operating conditions than in the Howell design. This constitutes the advantage of the Obry steering device.

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Section 15. Gyro Indicator of Longitudinal Tilt of an Aircraft and of Deviations

From its Course*

Use of gyroscopes is particularly widespread in modern aircraft instruments. Most of the instruments that make "blind" flying of aircraft possible, i.e., flight in absence of visible ground marks (darkness of night or clouds), are based on the application of the gyroscope. A few applications of the astatic gyroscope in instruments for blind flying will be discussed below.

Under conditions of instrument flying, when the pilot has no opportunity to judge the position of the aircraft by observing ground marks, he needs an instrument to provide indications of every deviation of the aircraft from correct rectilinear flight in the assigned direction; in other words, he needs an instrument that will immediately warn him of any deviation of the aircraft from the set course, or of any deviation from the horizontal direction of flight (going into a dive or climbing), so that the pilot, by manipulating the control surfaces, is in a position to rectify the flight of the aircraft. It is easy to construct the scheme of such an instrument, using the properties of the astatic gyro with three degrees of freedom.

Let us imagine that the instrument panel in the cabin in front of the pilot includes an instrument with a box-shaped body, containing an astatic gyro in Cardanic suspension, with three degrees of freedom (Fig.38). The axis of rotation of the outer ring a lies in bearings rigidly attached to the body of the instrument and directed horizontally, transverse to the body of the aircraft; the axis of rotation yy of the ring, in the normal position of the instrument, is directed vertically; the axis of rotation zz of the gyro rotor P, in the normal position of the instrument, is directed horizontally along the longitudinal axis of the aircraft.

The inner ring b, on the extension of the gyro axis, is rigidly connected with the white disk c. The wall of the instrument, facing the pilot (in Fig.38, the right

* Translator's note: In U.S. terminology, a turn-and-bank indicator.

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wall) is covered with a glass plate on which the circle d is blackened, covering the white disk c. The rotor of the gyro is placed in rapid rotation.

If the aircraft is in rectilinear horizontal flight in the assigned direction,
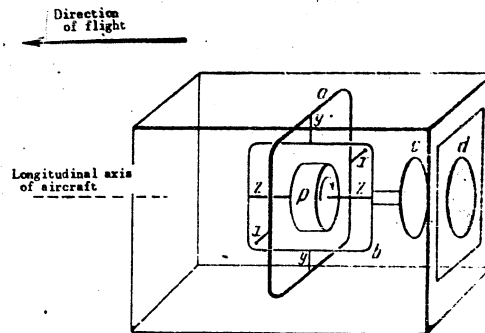

Fig.38

then the white disk c is covered by the black circle d, and is not seen by the pilot.

Let us assume that the airplane deviates from a horizontal flight; assume that it
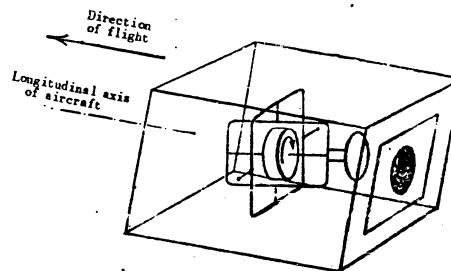

Fig.39

begins to gain altitude (changes to a "nose-up" attitude) and its front end is elevated above its tail. Our instrument, rigidly attached to the body of the aircraft, will likewise loose its horizontal position (Fig.39), while the gyroscope in the in-

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strument stably maintains the original direction of its axis, and still remains horizontal. It will be seen from Fig.39 that now the black circle on the glass of the instrument no longer completely covers the white disk. To the pilot, looking at the instrument, part of the white disk appears as having emerged from beneath the black circle d and moved on the glass plate of the instrument (Fig.40). If the aircraft goes into a "dive", the instrument will indicate this by the fact that the white disk emerges from the black circle on the lower side. It is easy to imagine that in the same way, the instrument will detect any deviation of the aircraft from the assigned direction of flight. At a deviation from the assigned direction toward the right, the white disk will appear from beneath the black circle to the right; its
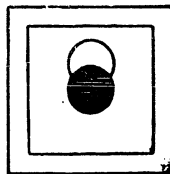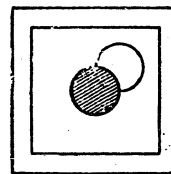
Fig.40   Fig.41

appearance to the left of the black disk indicates a leftward deviation of the aircraft from the set course. By observing the instrument, the pilot is thus able to check the accuracy of flight. For example, if he sees that the white disk appears to the right of, and below, the black circle (Fig.41), he will know that the aircraft is deviating to the right of the set course and has gone into a dive. By manipulating the rudders the pilot can correct the position of the aircraft, even without ground view.

The gyroscopic turn-and-bank of the above type was designed by the British designer of gyro instruments, Brown (Bibl.1)

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Section 16.   The Gyroscopic Semi-Compass*

Another instrument based on the same idea is widely used in aviation under the name gyro semi-compass. The operating principle of this instrument will be discussed below.

Naturally, it is readily possible to keep an aircraft on its course in the absence of visible landmarks by means of an ordinary magnetic compass. However, operation of the magnetic compass on board an aircraft is very difficult, because of the constant fluctuations in the position of the instrument under flying conditions. A more stable directional indicator for aircraft, by which the course can be determined, is required. It is natural enough to think of using a rapidly rotating astatic gyro with three degrees of freedom, possessing a high degree of stability, as such a course indicator.

Let us assume an aircraft to be in horizontal rectilinear flight in the assigned direction AB, which is called the course of the aircraft (Fig.42a). To avoid complicating the matter we assume that the flight is made in windless weather (there is no difficulty in allowing for the influence of the wind), and mark the direction of the meridan SN (S,south; N,north). The angle γ between the directions SN and AB is termed the course angle or heading of the aircraft; this angle is measured from north to east (so that, e.g., a course angle of 45° corresponds to a direction of flight toward northeast, while a course angle of 270° corresponds to a flight toward west). The pilot's task is to maintain the prescribed heading.

We now assume that the pilot's cabin contains a rapidly rotating astatic gyro in Cardanic suspension with three degrees of freedom, of the design indicated in Fig.2; the axis of rotation in the outer ring of the gyro is set vertically (the outer ring is not shown in Fig.42). If we set the axis of the gyro rotor horizontally in the

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* Translator's note: In U.S. terminology, a gyro compass.

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direction of the meridan SN, the angle between the rotor axis and the longitudinal axis of the aircraft will give the heading γ.

We already know that a rapidly rotating astatic gyro with three degrees of freedom has the property of stably maintaining a constant direction of its axis in space. If the earth did not rotate, the axis of such a gyro would also maintain its constant direction with respect to the ground. Let us disregard the earth's rotation, for
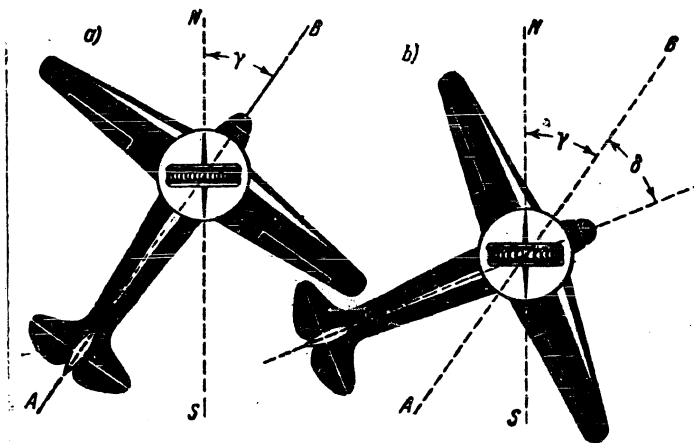

Fig. 42

the time being. If there were no diurnal rotation of the earth, the axis of our gyroscope, once set in motion, would remain directed along the meridan without change, and any variation in the heading of the aircraft would result in the same variation of the angles between the gyro axis and the longitudinal axis of the aircraft. Let us assume that the aircraft deviates from its course and the heading varies by the quantity (Fig. 42b). This would immediately be detectable from the corresponding variation in the angle between the gyro axis and the longitudinal axis of the aircraft. The pilot must deflect the rudder and return this angle between the axes of of the aircraft to its original value γ. In this way the pilot could maintain the heading of the aircraft without using landmarks or a magnetic compass.

This would be the situation if the earth did not rotate. Let us now define the

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complication introduced by the diurnal rotation of the earth in the operation of the instrument.

To take the simplest case, let us put ourselves into the position of the crew of the airplanes that left I.D.Papanin and his companions at the North Pole. At the North Pole all meridian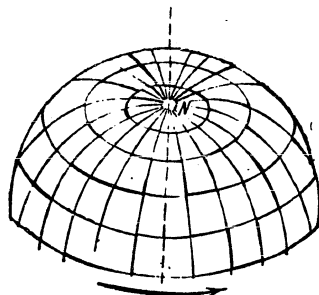s converge in a single point (Fig.43); all of them rotate together with the earth about the axis of rotation of the earth, which passes through the North and South Poles; they rotate from right to left (counterclockwise), making one revolution per day. During the period of one hour, each meridian rotates by $\frac{360^o}{24} = 15^o$. If the axis of a gyroscope, installed on an aircraft flying near the North Pole, is to maintain the position of the meridian without change, the gyro must be so arranged that the gyro axis no longer maintains a stable direction in space but rotates uniformly in a horizontal plane from right to left, rotating by 15° during the course of each hour.

Fig.43

We know already that this can be done, since the phenomenon of precession of the gyro, with which we are familiar, is involved here. To induce a precession of the gyro in the horizontal plane, it is sufficient to apply an appropriate vertical force to the gyro axis. Let us attach a counterpoise weighing p grams to the inner ring, along the extension of the rotor axis and precisely at that end of the rotor axis from which the proper rotation of the rotor appears to be counterclockwise. According to the above-described rule of precession (cf. Section 6), it is easily found that, under the action of the force p, the gyro will precess in a horizontal plane, rotating counterclockwise, i.e., from right to left, about a vertical axis.

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The angular velocity of precession is expressed by eq.(4):

$$w_1 = \frac{pa}{J\omega}$$

where J is the moment of inertia of the rotor, w the angular velocity of its proper rotation, and a the distance between the point of application of the force p and the point of intersection of the Cardanic axes. The instrument provides means for varying the value of a. Obviously, the value of a can be so selected that the angular velocity of precession of the gyro will have the value we require, corresponding
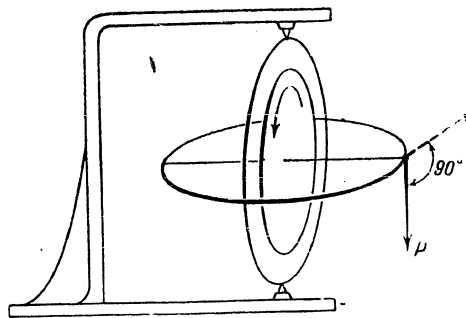

Fig. 44

to the rotation of the gyro axis through 15° during the course of one hour. This operation constitutes the adjustment of the instrument.

We have considered the operation of the instrument in the region of the North Pole. On the whole, the situation is the same at other latitudes, with the single difference that, in the middle latitudes, the plane of the horizon rotates about the vertical and turns, during one hour, not through 15° but through a smaller angle*.

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* This question will be discussed further in Section 22. There it will be demonstrated that, at all points of the earth's surface except the poles, the plane of the horizon rotates not only about the vertical but also about the meridian. This fact is partly responsible for the phenomenon of the rising and setting of heavenly bodies.

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For example, at the latitude of Leningrad, the plane of the horizon rotates through approximately 13° per hour. Of course, the instrument must be so adjusted to this angular velocity of precession that, at the latitude of Leningrad, the axis of the gyro maintains the direction of the meridian without change.

This is the principle of action of the gyroscopic course indicator, also known as gyroscopic semi-compass. Obviously, this instrument is not a compass in the full sense of the word. It cannot completely replace the conventional magnetic compass. The gyro axis in this instrument does not possess the power of aligning itself automatically along the meridian as does the magnetic needle of a regular compass. Intervention by the observer is required to align the gyro axis with the meridian. An important property of the instrument is that it possesses the power of maintaining the direction of the meridian with great stability, once it is set. In this lies its immense superiority to the ordinary compass.

Section 17. Design Features and Operation Details of the Gyroscopic Semi-Compass

Having discussed the operating principle of the gyroscopic semi-compass, let us now give a few details of its design and operation.

The instrument is mounted in a hermetically sealed body (box) which is attached to the instrument panel in front of the pilot's seat in the aircraft cabin. The axis of rotation of the outer ring of the gyro is arranged vertically (both rings in our instrument are constructed in the form of rectangular frames which, of course, is not a point of substantial importance). The pilot follows the apparent displacements of the instrument in the horizontal plane by observing, through a window in the wall of the body, the displacement of a horizontal graduated circle attached to the outer ring of the instrument (the outer frame). The wall of the instrument body (on the glass of the window) is provided with a so-called course mark, which corresponds to the direction of the longitudinal axis of the aircraft. The division of the graduated circle directly opposite the course mark gives the heading of the aircraft.

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Figure 45 gives an external view of the instrument from the side facing the pilot: a is a round window; the field b beyond the window is covered by an opaque mask with a rectangular opening through which part of the graduated circle can be seen; and cc is the course mark.
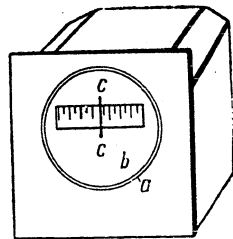
For correct operation of the instrument it is very important that a constant rotational speed of the gyro rotor be maintained for a long time. In the instrument we are describing this is done in the following way:

The air is continuously aspirated from the body of the instrument by means of a Venturi tube, thus producing a pressure differential within and without the body. Under the action of this pressure difference, the outer air is drawn into the body of the instrument through an opening in the bottom of the body, and a jet of air enters the channel a (Fig.46) cut into the step bearing the outer ring b (outer frame). From there, the air jet enters the nozzle c, which is rigidly connected with the outer frame b. The rim of the rotor d, whose axis e is attached to the inner frame f, is provided with grooved slots. The air jet impinges with great force on these slots on issuing from the nozzle c; this ensures uniform rotation of the gyro rotor. The air pressure in the body is brought to 90 mm Hg; in this case the gyro rotor runs at about 12,000 rpm. The direction of air circulation within the casing is indicated in Fig.46 by arrows.

There is still another interesting design feature of the instrument, by means of which the gyro axis is automatically returned to a horizontal position (or more exactly, into a position perpendicular to the axis of rotation of the outer frame),

Fig.45

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if for any reason it leaves that position*.

The rim of the rotor is provided with two ridges for directing the air jet issuing from the nozzle a which, in turn, is attached to the outer frame b (Fig.47).
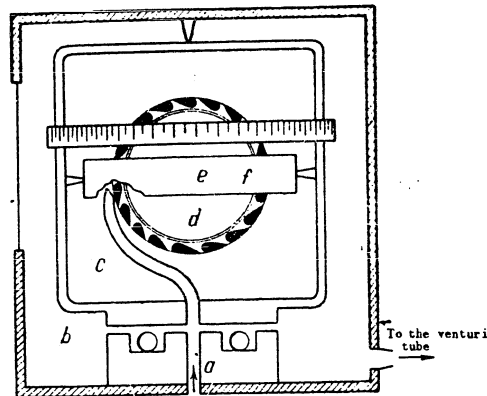

Fig.46

Let us imagine that, for any reason, the axis of the rotor AB has left the horizontal position; let us assume that the end A of the axis is lowered and the end B is raised (Fig.48). Now the air jet will strike not only the slots of the rotor but

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* We remark that the axis of an ideal free gyroscope with three degrees of freedom always has a tendency to leave the plane of the horizon. We recall (Section 11) that the axis of such a gyro possesses the property of maintaining a constant direction in space, i.e., a constant direction relative to any star and, consequently, has a tendency while following that star to rise above the plane of the horizon in the eastern half of the sky and to descend toward the horizon in the western half of the sky. This is due to the rotation of the plane of the horizon about the meridian line, mentioned in a previous footnote and to be discussed in more detail in Section 22.

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also against right-hand ridge; the pressure of the jet will yield not only a compo-
nent directed along the tangent to the rim of the rotor, imparting to it a rotation
about the axis AB, but also a component perpendicular to the ridge. This component
tends to rotate the entire instrument about the vertical axis of rotation zz of the
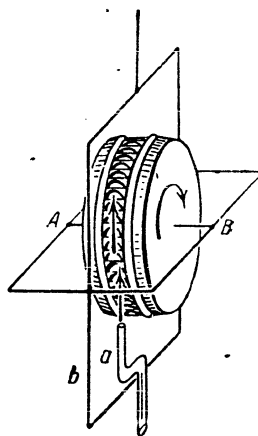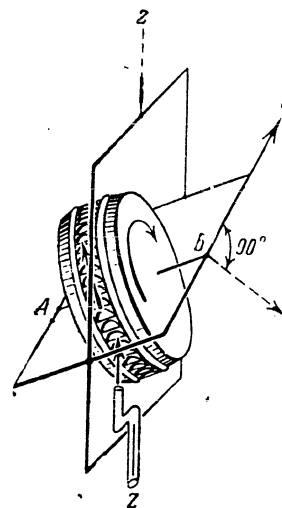outer frame (counterclockwise). The pressure component of the jet, normal to the

Fig.47   Fig.48

ridge, is transmitted to the axis AB of the rotor, and may be replaced by a force
equivalent to it (which we shall term F), applied at any point of the axis AB (e.g.,
at the point B) and directed horizontally and normal to the axis AB. We know that
such a horizontal force must lead to a precession of the gyro axis in the vertical
plane. By applying the known rule of precession to the force F and allowing for the
sense of rotation of the rotor about the axis AB, imparted to it by the air jet (this
sense is indicated in Figs.47 and 48 by arrows), it is easy to demonstrate that the

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precession of the rotor axis due to the pressure of the air jet on the right-hand ridge will lower the B end and raise the A end of this axis, i.e., will return it to the horizontal position.

In this way, the automatic restoration of the rotor axis to the plane of the horizon is ensured. When the rotor axis deflects from this plane in the opposite direction, this deviation will in turn be counteracted by the pressure of the air jet on the left-hand ridge.

An ideally constructed and perfectly regulated instrument of this design could maintain its meridional direction, once given, for an indefinite period. However, the imperfections of the actual construction limit the period of accurate operation of the instrument. After a certain length of time, usually 15-20 min, the instrument deviates markedly from the direction of the meridian which was originally assigned to it, and then must be manually reset to the meridian, by comparing its readings with the readings of a conventional magnetic compass. The instrument is considered in satisfactory operating order if this deviation does not exceed 3° after 15 min.

According to I.T.Spirin, former navigator of the leader aircraft of the expedition that left the I.D.Papanin group at the North Pole, the gyroscopic semi-compass is an irreplacable instrument for flights in the Arctic (Bibl.2). In the region of the Pole, where magnetic compasses are useless, the navigator has only the gyroscopic semi-compass at his disposition; its readings can be checked by astronomic and radio-physical methods. At a particularly careful adjustment of this instrument, its readings can be used for a considerably longer period than indicated above (Bibl.3).

The gyroscopic semi-compass of the above design was first constructed by the famous American gyro instrument firm of Sperry; at present, it is widely used in USSR aviation. There are also other designs of the gyroscopic semi-compass, in which the astatic gyro with three degrees of freedom is also used as turn indicator.

The German Anschuetz gyro compass differs from the above-described instrument in

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that the rotation of the gyro rotor is produced by means of electric power instead of by the pressure of an air jet. The gyro rotor is built like the rotor of an electric motor; the axis of the stator of this electric motor is mounted directly to the inner ring of the instrument; the gyro rotor freely rotates about this axis. The rotor in this instrument makes about 20,000 rpm. The instrument has no device for automatically aligning the gyro axis with the plane of the horizon. The accuracy of its readings is about the same as that of the Sperry type gyro compass.

The gyro compass of the British builder of gyroscopic instruments, Brown, is considerably more accurate (Bibl.4). The high precision of this instrument is explained primarily by the replacement of conventional ball bearings with greatly improved bearings of special design. The gyro axis is automatically brought into the plane of the horizon, in this instrument, by an original device based on the reaction of an air jet produced by the rapid rotation of the gyro rotor, acting like a rapidly rotating fan. The instrument was manufactured by Brown in two models: the lighter type for aircraft and the heavier for operation on a ship, in the fire-control system of naval artillery.

Section 18. Aircraft Turn Indicator

The gyro compass allows the pilot to determine the course of the aircraft at any instant: by following the readings of the gyro compass for a certain length of time, he is able to determine whether the aircraft is flying in a straight line or is deviating from this straight line in any direction, to one side or the other. There is another instrument used in aviation practice which directly measures the angular velocity with which an aircraft is turning. One glance at this instrument is sufficient for the pilot to determine, under conditions of blind flying, whether the aircraft is flying in a straight line or not; and not only this, but the instrument directly gives the rate of turn of the aircraft, if it is flying along the arc of a curved line. This instrument is also based on using the properties of a rapidly

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rotating gyro and is known as gyroscopic turn indicator.

In our considerations of the technical applications of the gyroscope, up to now, we have been speaking of a gyro with three degrees of freedom; its fundamental prop-erty, namely the stability imparted to it by rapid rotation, was the basis for the instruments we have thus far considered. In the turn indicator, however, we meet a technical application of the gyroscope with two degrees of freedom. This instru-ment utilizes the remarkable property of a gyroscope with two degrees of freedom which becomes manifest when it is placed on a rotating base.

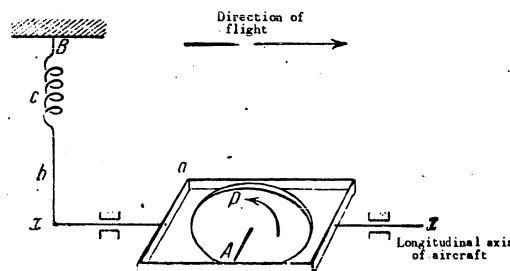In Section 12 we considered the question of the behavior of a gyro with two de-

Fig.49

grees of freedom when placed on a rotating base. We saw there that the behavior of such a gyro is determined by the rule which we termed Foucault's rule and which states that the axis of a gyro with two degrees of freedom, placed on a rotating base, tends to corresponding parallelism with the axis of rotation of the base. We recall here that two parallel axes of rotation are, by convention, termed corre-spondingly parallel in the case when the rotation about these axes is in one and the same sense.

Let us now imagine that a rapidly rotating gyro with two degrees of freedom is installed in the pilot's cabin (Fig.49). The axis of rotation of the rotor of the

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gyroscope P is attached to the frame a which, in turn, is free to rotate about the

xx axis, whose bearings are rigidly connected with the body of the instrument and,

consequently, with the aircraft. This xx axis runs horizontally along the longitu-

dinal axis of the aircraft; the axis of rotation of the gyro rotor, in the normal

position of the instrument, is likewise directed horizontally, but perpendicular to

the longitudinal axis of the aircraft. To the xx axis of the frame a the indicator

b is rigidly attached which, in the normal position of the instrument, points in a

vertical direction; this indicator is connected with the helical spring c whose oth-

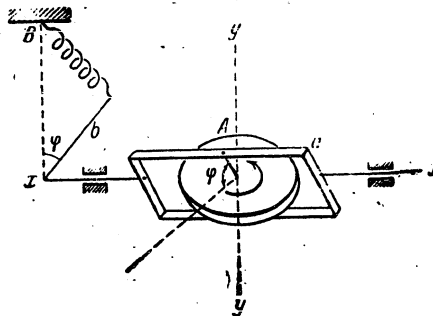er end is attached, at the point B, to the body of the instrument. It is obvious

Fig.50

that, in this case, the gyro has two degrees of freedom, one of which corresponds to

the proper rotation of the rotor and the other to the rotation of the frame a about

the xx axis. The proper rotation of the rotor is maintained in this instrument by

means of an air jet, on the same principle as that used in the gyro compass.

Let us assume that the aircraft makes a turn to the left; for simplicity, we

also assume that this turn is accomplished without banking (in reality every correct

turn of an aircraft is accompanied by a bank; in a turn to the left, the left wing

rises and the right wing dips). Under these conditions, the aircraft is no longer

in pure translational motion, since it acquires simultaneously a rotary motion in

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the horizontal plane, i.e., about a vertical axis. The angular velocity of this rotary motion of the aircraft is higher the sharper the turn made by the aircraft.

Our gyro, installed on the aircraft, is in the position of a gyro placed on a rotating base. Since the gyro has two degrees of freedom, the properties discussed in Section 12, which we have just recalled, must be manifested. According to Foucault's rule, the axis of rotation of a gyro rotor must show a tendency to corresponding parallelism with the vertical axis of rotation yy of the instrument base (i.e., the aircraft itself); (Fig.50).

Let us assume that the proper rotation of the gyro rotor is counterclockwise if viewed from the A end of its axis, as shown by the curved arrow in Fig.49. The rotation of the aircraft about the vertical axis yy, when it makes a left turn, likewise appears to be counterclockwise, if the airplane is viewed from above. According to Foucault's rule, the axis of the gyro rotor will tend to coincide with the vertical axis yy of rotation of the aircraft. Consequently, the axis of rotation of the gyro will turn, together with the frame a, about the xx axis in its effort to coincide with the vertical axis yy, and this rotation of the rotor axis will take place in a sense such that when it is completed, after coincidence of the rotor axis with the vertical axis yy, the proper rotation of the rotor will agree in sense with the rotation of the aircraft (i.e., will be counterclockwise if viewed from above). This means that the rotor axis will rise at its A end and will drop at its opposite end (Fig.50).

However, when the frame a rotates about the xx axis, the pointer b, connected with the frame a, will rotate about the same axis through the same angle. The pointer b will now exert traction on the helical spring connecting it with the fixed point B. To this tension the helical spring will oppose resistance; for this reason, the rotor axis will be unable to turn together with the frame a through a full $90°$, and will be unable to coincide with the vertical axis yy; rather, this axis will turn together with the frame through an angle φ (Fig.50) such that the resilience

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of the helical spring will be in equilibrium with the force tending to rotate the axis of the rotor to its coincidence with the vertical axis yy.

The sharper the turn made by the aircraft, the higher will be the angular velocity of its turn and the greater will be the force tending to bring the rotor axis of the gyroscope closer to the vertical axis of rotation of the aircraft yy; Consequently, the greater will be the angle φ through which the frame a is rotated together with the pointer b.

The instrument is placed in the pilot's cabin on the instrument panel before him. When looking at the instrument in the direction of flight, the pilot (if the aircraft is turning to the left) will see the pointer b deviate from its normal vertical position toward the left through the angle φ (Fig.50), which angle will be larger, the greater the angular velocity of the turn. In rectilinear flight, the pointer b remains in its neutral vertical position. It is easy to demonstrate that, during a right turn of the aircraft, the pointer will deviate from its neutral position toward the right.

Thus, the turn indicator makes it possible for the pilot to determine, under instrument flying conditions, whether the aircraft is flying in a straight line or is turning in some direction or other. The sense of deflection of the pointer b from its neutral position indicates the sense of the aircraft turn; the magnitude of this deviation permits estimating the magnitude of the angular velocity of turn by the aircraft.

To avoid complicating the discussion, we have assumed that the turn of the aircraft takes place without a banking. A bank which, in reality, always accompanies a turn of the aircraft, introduces no substantial modifications in the operating conditions of the turn indicator.

Section 19. The Gyroscope Monorail Car

The first years of the Twentieth Century were an era when various technical ap-

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plications of the gyro first became widespread and won general recognition. In these years, for the first time, a gyro compass suitable for shipboard use was developed which since then has taken firm hold in the navies of the entire world. In these same years, inventive thought centered on using the gyroscope for stabilization purposes.

The term gyroscopic stabilizers means devices having a double function. Used in systems which are inherently stable but are free to oscillate about their equilibrium positions, stabilizers have the function of damping the oscillations excited in the system by some factor. Here the gyro stabilizer appears in the role of an oscillation damper of the system. Such a stabilizer may also be used in a system which is not inherently stable. In such a case, the gyro stabilizer has the function of imparting stability to a system which in itself is unstable. The simplest example of such a case of gyroscopic stabilization is the simple top which is inherently unstable and only becomes stable in rapid rotation. In the present Section, this case of gyroscopic stabilization will be discussed in more detail.

At the beginning of the Twentieth Century attempts were made in various countries to design a monorail gyro-stabilized railroad. Up to that time, the bicycle had found wide use everywhere as an extremely convenient and economical means of transportation. The bicycle, invented in the Nineteenth Century, is per se unstable; it is artificially stabilized during a sufficiently rapid motion. It is well known to everyone that, with a certain amount of practice, the riding of a bicycle involves no particular difficulty; in motion, even at moderate speed, the stability of the bicycle remains entirely adequate.

How can this, at first glance surprising, stability of the bicycle when controlled by a sufficiently skilled rider, be explained? Does this case constitute an example of gyroscopic stabilization? That is, is it a question of stability imparted by rapidly rotating masses, in this particular case, by the two rapidly rotating wheels of the bicycle? This question must be answered in the negative. We

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do not have to do with the stabilizing action of the rotating wheels of the bicycle. Those of our readers who are able to ride a bicycle are well aware that the stability of the bicycle is maintained by the rider by means of slight motions of the handlebars. When the rider notices that the bicycle is beginning to deviate to the right, he immediately turns the handlebars slightly to the right and thereby forces the bicycle to move not along a straight line but along a curved one, whose center of curvature is on the right. The motion of the bicycle along the curved line instantly causes the appearance of a centrifugal force directed from the center of curvature, i.e., from right to left; this force, directed toward the left, is what then corrects the position of the bicycle that had begun to deviate to the right. On the other hand, when the bicycle deviates to the left, it is sufficient to turn the handlebar slightly to the left in order to cause an instant generation of a centrifugal force directed to the right, which then corrects the position of the bicycle. In this way the bicycle moves, not strictly along a straight line but along a slightly sinuous line, deviating from a rectilinear path, now to the right and now to the left. With a little practice, the turns of the handlebar necessary for maintaining stability are executed by the rider instinctively and automatically without participation of his conscious will. It is in acquiring the automatic habit of these motions that the mastery of the art of bicycle-riding consists*.

Of course, this principle of stabilization of a bicycle cannot be employed to impart stability to the car of a monorail railroad. The rail on which such a car

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* Readers acquainted with the principle of theoretical mechanics, of course, beware of the fact that this explanation of the stability of a bicycle (first given by Rankine) is based on the "method of kineto-statics" known in mechanics, which reduces the problem of motion of any system to the problem of its equilibrium by introduction of so-called forces of inertia, to which centrifugal force also belongs.

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rolls fixes the path of the car in advance and deprives the car of the freedom in any way to make any deviations whatever from its path; yet the possibility of such deviations is substantially necessary, as we have seen, for application of the principle on which a bicycle is stabilized.

However, it should be possible to stabilize a monorail car by a purely gyroscopic method, namely by installing a rapidly rotating and powerful gyroscope in the car. First, it must be determined whether this object can be reached by fixing the
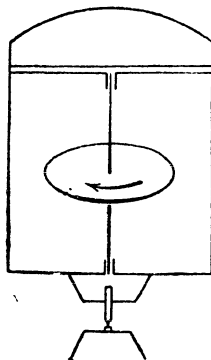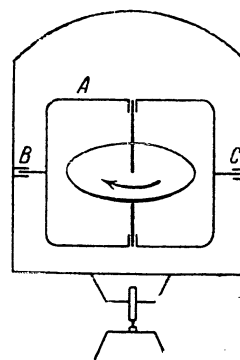
Fig.51   Fig.52

axis of rotation of a rapidly rotating and powerful rotor in a constant position with respect to the body of the car, e.g., as shown in Fig.51. The reader who has attentively followed our discussions in Chapter 1 will doubt this. The gyroscope depicted in Fig.51 has only two degrees of freedom (corresponding to its proper rotation and its rotation together with the car body about the rail axis), while the presence of three degrees of freedom is necessary for stability of the gyroscope. No matter how rapidly the rotor rotates, with an arrangement corresponding to that in Fig.51, this rotation will not increase the stability of the car to the slightest extent.

In order to render the car stable, the set-up of the stabilizer must be modified by giving the gyroscope its missing third degree of freedom. Let us assume the axis of the rotor to be attached in the frame a, which itself is free to rotate about the axis BC, located transversely in the car body (Fig.52). Here the gyroscope already has three degrees of freedom (corresponding to its own proper rotation, rotation together with the frame A about the axis BC, and rotation together with the car body about the rail axis).

It is easy to see that the scheme shown in Fig.52 is sufficiently close to the scheme of the gyroscope in Cardanic suspension with three degrees of freedom; the frame A here plays the role of the inner ring of the suspension, while the car body plays the role of the outer ring. Of course, it is impossible to speak here of an astatic gyro or to apply to the scheme of Fig.52 the reasoning as to the stability of the gyro discussed in Section 7.

Let us assume that the car of a monorail railroad is equipped with a gyroscope of the type shown in Fig.52 and that, in the normal position of the car, the axis of the gyro rotor is directed vertically. Let us assume that the car is not in translatory motion along the rail but that the rotor is maintained in a state of rapid rotation (e.g., clockwise, if viewed from above, as shown in Fig.52). Let us now assume that the car begins to tilt to the right, threatening to lose its equilibrium and tip over to the right. How can the gyroscope keep the car from falling?

It will be demonstrated below that the gyro performs this function if we are able, in some way or other, to impart to its frame A a sharp rotary motion about the transverse axis BC, in a counterclockwise sense if viewed from the right (Fig.53). Now let us analyze what forces will act in this case, on the car body, exerted by the gyro frame.

The rotation of the frame A about the axis BC corresponds to the precessional rotation of the gyro axis about the axis BC. Any precessional rotation of the gyro axis, however, as we have seen in Section 5, assumes the existence of a certain

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couple of forces applied to the gyro axis, causing the precession of the gyro.
These forces can be applied to the gyro axis at the point where it makes contact
with external bodies, i.e., the supporting bearings attached to the frame A; this is
nothing other than the reaction of the bearing applied to the ends of the gyro axis.
To determine the direction of these forces, the rule of precession formulated in
Section 6 can be used. By imparting to the frame A a rotation in the indicated sense
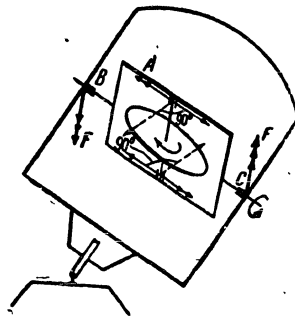about the axis BC, the points of the gyro axis will be given a displacement normal

to the plane of the frame; in this
case, the upper end of the gyro
axis will be displaced toward the
reader, and the lower end away
from the reader; the directions of
these displacements are indicated
in Fig.53 by broken arrows. By
rotating these arrows about the
gyro axis through 90° in a direc-
tion opposite to that of the prop-
er rotation of the gyro rotor, the
direction of the forces applied to

Fig.53

the ends of the gyro axis by their bearings can be defined. These forces are indi-
cated in Fig.53 by the solid arrows.

However, action equals reaction. If the ends of the gyro axis are subjected to
the above forces, exerted by the bearings supporting these axes, then forces of the
same magnitude but opposite in direction must be exerted on these bearings by the
gyro axis; in Fig.53 the forces applied to the bearings by the gyro axis, i.e., the
forces with which the gyro axis presses on its bearings are indicated by the solid
double-headed arrows. Through the frame A, these forces are transmitted to the
bearings supporting the ends of the axis BC and attached to the car body. In this

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...y, the frame of the gyro transmits to the car body forces (denoted by the symbol F) generated by the bearings of the axis BC and shown in Fig.53 by the solid triple-headed arrows.

These forces F form a couple tending to rotate the car body counterclockwise (from right to left) and counteracting the force of gravity, which tends to overturn the car from left to right. The sharper the precessional motion imparted to the frame of the gyro A, the greater will be the forces F. If their value is sufficiently high, they will cancel the overturning action of the force of gravity on the car and will return it to a vertical position, re-establishing its equilibrium.
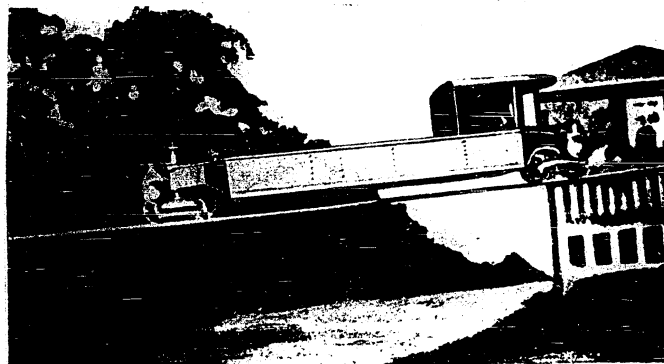

Fig.54

It is, of course, obvious that when the car threatens to tip toward the left, it may likewise be kept from overturning by imparting to the gyro frame A a sharp precessional motion in the opposite direction, which would cause generation of the couple of forces F in the opposite direction.

Such a purely gyroscopic method could assure the stability of the car on a monorail railroad. The whole problem reduces down to impart to the axis of the gyro in the car, at the instant of loss of equilibrium, a series of sharp precessional

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motions toward one side or the other necessary to restore equilibrium. Of course, these precessional motions of the gyro frame must be entirely automatic in operation. The entire difficulty rests in the design of the corresponding automatic device.

As mentioned before, at the beginning of the Twenticth Century this problem attracted the attention of inventors in various countries. The solutions proposed by them differ in the design of the automatic device for effecting the necessary precessional motions of the gyro frame. The first design for a monorail gyroscopic railroad appeared in England. It was invented by Brennan. The photograph in Fig.54 shows a small model of the Brennan monorail gyro-stabilized car, maintaining its stable equi-
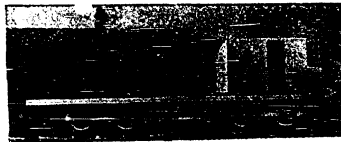

Fig.55

librium on a stretched wire; the same car, able to carry up to 40 passengers and used for transporting the visitors to an exposition over the fair grounds, is shown in another photograph in Fig.55. The Brennan invention, however, failed to pass the experimental stage and was never utilized in practice for commercial purposes. Similar attempts in Germany and the USSR, where an experimental monorail line of a gyro-stabilized railroad from Lenigrad to Gatchina was planned at the beginning of the 1920's, likewise ended in failure.

Section 20. Stability of a Rotating Projectile in Flight

In concluding this Chapter, an interesting application of the laws of gyroscopic phenomena to warfare will be discussed. Since the time when spherical projectiles in rifle and artillery practice were replaced by projectiles of elongated shape, it became clear that, in order to increase the accuracy of fire, such projectiles had to be given stability in flight; for this purpose, it became necessary to give them

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a rapid rotation about their longitudinal axis. This led to the introduction of the rifled weapon in both infantry and artillery. The ball or projectile leaving a barrel with a helical rifling receives a rapid spin about its longitudinal axis and thereby acquires all the properties of a rapidly rotating gyro. Let us see how the laws of known gyroscopic phenomena are manifested in the flight of an elongated projectile.

If the flight of a rotating projectile would take place in an airless space (or if the flight of the projectile would take place in the stratosphere at a height of 20 km or more, where the density of the air is very low)*, then a rotating projec-
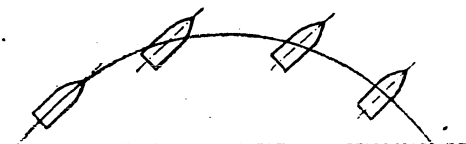

Fig.56

tile, owing to its gyroscopic properties, could be compared to an astatic gyro with three degrees of freedom. Indeed, the only force acting on the projectile during its flight in this case would be the force of gravity which, applied at the center of gravity of the projectile, would affect the translatory motion of the projectile, but would exert no influence on its rotary motion about its center of gravity; the rotation of the projectile about its center of gravity would take place in all respects exactly as though the center of gravity of the projectile were rigidly fixed and no forces whatever were applied to the projectile. In this case, the rotating projectile would acquire the fundamental property of the astatic gyro with three

* An artillery projectile actually rises to such an altitude in very long-range firing.

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degrees of freedom, namely its power to maintain a constant direction of its axis of rotation. It is easy to show that, in this case, the direction of the longitudinal axis of the projectile could not coincide, throughout the entire time of flight, with the direction of the translatory motion of the projectile (Fig.56).

The situation is different in the flight of a rotating projectile through the air. In its motion through the atmosphere, the projectile is given the remarkable property of always flying with its head section forward (Fig.57). This is what is
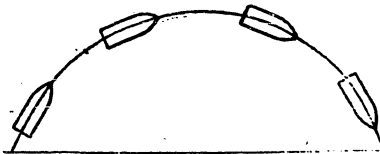

Fig.57

responsible for the high accuracy of fire from a rifled weapon. It now remains to show that this property of a rotating projectile is a consequence of the resistance of the air encountered by the projectile in its flight through the atmosphere.

Let us consider the conditions of flight of a projectile through the atmosphere.

To interpret the power of resistance of the air acting on a flying projectile, let us assume at first that the air stream flows around a projectile immovably fixed in its path (wind-tunnel experiments in aerodynamics laboratories are staged in this way). Figure 58 shows a model of a projectile placed in an air stream. The air flowing around this model produces, at various points of its surface, pressures that are compounded into a single resultant R which is termed the force of resistance of the air acting on the projectile model. If the longitudinal axis of the projectile is exactly directed against the air stream, the direction of the resistance force R exactly coincides with the direction of the air stream, i.e., with the direction of the longitudinal axis of the projectile. However, if the longitudinal axis of the

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projectile deviates somewhat from the direction of the air stream, then, as shown

by research, the direction of the force R likewise deviates from this direction; in

this case, the line of action of the force R intersects the longitudinal axis of the

projectile in its front section, i.e., in front of the center of gravity of the pro-

jectile (Fig.58; here the center of gravity of the projectile R is denoted by c.g.).

The same distribution of aerodynamic forces takes place in the flight of a projectile
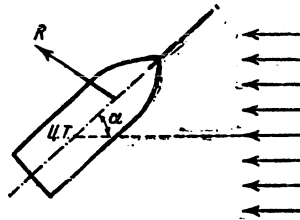
through a quiet atmosphere.

Let us imagine that the follow-
ing experiment is staged in an
aerodynamic laboratory: A model of
an elongated projectile is placed
in an air stream in such a way
that the center of gravity of the
model cannot be displaced, but that
the model is free to rotate about

Fig.58

its center of gravity. We assume further that the model of the projectile is given

no rotary motion about its longitudinal axis. It is clear that, if the longitudinal

axis of the projectile forms a certain angle α with the direction of the air stream

(Fig.58), then the action of the force of air resistance will cause the model to ro-

tate about its center of gravity in the sense of increasing angles α , and the po-

sition of the model relative to the angle $\alpha = 0$ will be unstable.

Thus, under the action of the air resistance, the elongated body of a projectile

placed in an air stream has a tendency to deviate with its longitudinal axis from

the direction of the air stream in the sense of increasing angles α . For this

same reason, a nonrotating elongated projectile, in its flight through a quiet atmos-

phere, proves to be unstable in flight. Thence the tendency of such projectiles to

overturn and somersault in flight, which strongly reduces the accuracy of fire with

such projectiles.

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Returning to our experiment in the aerodynamics laboratory, we now assume that the model of the projectile placed in the air stream is given a rapid rotation about its longitudinal axis, which rotation is clockwise if viewed from the tail section of the model (Fig.59); we assume again that the longitudinal axis of the model forms a certain angle α with the direction of the air stream. The rapid rotation will now impart to our model the properties of a rapidly rotating gyroscope. The action of the air resistance R can then be found from the known rule of precession.

Proceeding as in Section 6, we now resolve the force R into two components R_1 and R_2 of which the latter is directed along the longitudinal axis of the model and
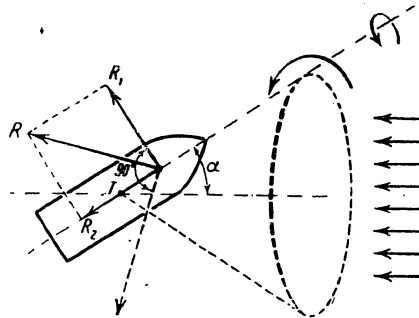

Fig.59

the former is normal to that axis. The component R_2 is balanced by the reaction of the fixed center of gravity of the model, while the component R_1 now causes a precession of the longitudinal axis of the model. In order to determine the direction of this precession, let us rotate the direction of the force R_1 through 90° about the longitudinal axis of the model in the sense of rotation of the model. We shall see that the longitudinal axis of our model will "deflect" toward the observer, displacing its front end in a direction normal to the plane of the drawing in Fig.59. When the force R_1 acts continuously, the longitudinal axis will rotate about the

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direction of the air stream, describing a cone with its vertex at the immovable cen-
ter of gravity of the model (Fig.59) and will preserve the angle   formed with the
direction of the stream. If at the beginning of our experiment this angle was small,
the longitudinal axis of the rotating model of the projectile will remain close to
the direction of the stream during the entire experiment, and will rotate with a
conical motion about this direction. We shall see that the model of the rotating
projectile, placed in an air stream, loses its tendency to deviate with its longitu-
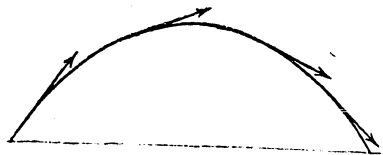dinal axis from the direction of the stream; conversely, if it is placed with its
vertex facing the air stream, it
will then acquire the ability of
maintaining a direction close to
the direction of the stream, by
describing around it a cone with
a small angle of divergence.

Fig.60

Let us now define the behavior
of a rotating projectile in its flight through a quiet atmosphere.

In flying through a quiescent air medium, the relative air flow will circulate
around the projectile, moving in a direction diametrically opposite to the motion of
the projectile, i.e., in the direction of the tangent to its trajectory. When the
projectile leaves the barrel of the weapon, the direction of motion of the projec-
tile, and likewise the direction of its longitudinal axis about which the rotation
takes place, will coincide with the direction of the axis of the gun barrel. During
the subsequent flight of the projectile, the direction of its motion gradually va-
ries, the tangent to its trajectory gradually drops (Fig.60) and, consequently, the
direction of the countercurrent of air flowing around the projectile varies gradu-
ally. If the flight of the projectile took place in a vacuum - or in the strato-
sphere- then the longitudinal axis of rotation of the projectile, as stated above,
would stably maintain its direction in space, i.e., would be displaced while always

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remaining parallel to its original direction (cf. Fig.56). In the flight of a projectile through the air, however, the air resistance becomes effective; under its action, the rotating projectile (as shown above) acquires the ability of stably maintaining its longitudinal axis of rotation in a direction close to the direction of the relative air stream or, what is the same thing, to the direction of motion of the projectile. In this way, under the action of the air resistance, the rotating projectile will actually acquire the ability of moving always with its nose section facing forward (cf. Fig.57),thus ensuring accuracy of fire with rotating projectiles. The longitudinal axis of rotation of the projectile acquires the ability, so to speak, of "following" the direction of its motion, i.e., following the direction of a tangent to its trajectory.

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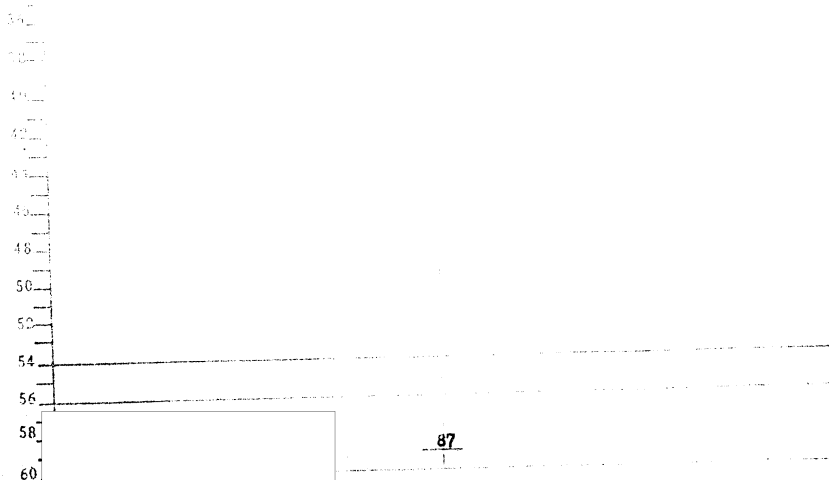
CHAPTER III

THE GYRO COMPASS

Section 21. Foucault's Original Concept

One of the most remarkable technical applications of the gyroscope is the gyro compass, widely used in the navies of the entire world. The original idea of using a gyroscope for a purely mechanical nonmagnetic compass was developed by Foucault as long ago as 1852. We mentioned above that, toward the end of the Nineteenth Century, the replacement of the magnetic compass by the mechanical compass on warships became mandatory, since proper functioning of an ordinary magnetic compass on a steel ship is practically impossible, owing to the various perturbations of the magnetic compass by the large iron masses on the ship and by the various electromagnetic influences exerted on the magnetic compass by the complex electrical equipment of a warship. Let us first analyze the essential nature of Foucault's original concept.

The earth is an immense magnet whose poles are located near the geographic poles of the earth. The magnetic field surrounding the earth acts on the magnetic needle placed in this field and aligns this needle with the direction of the magnetic meridian. This constitutes the operating principle of a magnetic compass.

However, it is known that the earth possesses another property: It rotates with a diurnal rotation about an axis (this axis is termed the celestial axis) passing through its geographic poles (the north N and the south S); the diurnal rotation of the earth is counterclockwise if it is viewed from the northern end of the celestial axis (Fig.61). It would be possible to utilize the fact of the diurnal rotation of

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the earth to devise a purely mechanical compass if we had a mechanical system as sensitive to the rotation of the base of this system as the magnetic needle is sensitive to terrestrial magnetism.

Foucault's experiment showed that a rapidly rotating gyro with two degrees of freedom is such a mechanical system; we know that such a gyro placed on a rotating base, reacts to the rotation of the base by having its axis tend to assume a position parallel to the axis of rotation of the base (according to the well-known Foucault's rule). This gives a possibility of utilizing the gyro with two degrees of freedom as a mechanical compass.
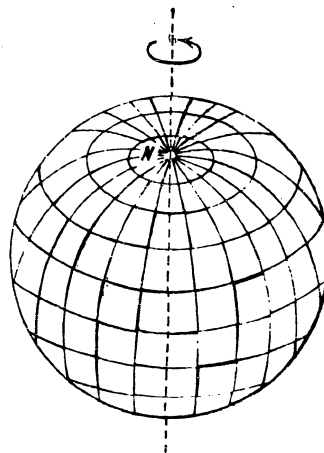
Let us imagine a gyro with two degrees of freedom (Fig.62). The axis of rotation of the rotor AB is attached to the frame a, which itself may rotate about the vertical axis xx. In all positions of the instrument, the axis AB remains horizontal. The proper rotation of the rotor is assumed to be counterclockwise when viewed by an observer from the A end of the axis AB (Fig.62).

Fig.61

Let us imagine the earth, and let us take some point M on its surface (Fig.63). Draw the earth's radius OM through this point; the extension Mz of the earth's radius OM will give the direction of the vertical at the point M, while the plane H, perpendicular to the vertical Mz, is the horizontal plane at the point M. Let us also draw the straight line ns, along which the horizontal plane H intersects the plane of the meridian NM passing through the point M; this straight

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line is termed the meridian at the point M.

Assume that the gyro with two degrees of freedom, represented in Fig.62, is placed on the horizontal plane H in such a way that the axis of rotation of the frame A is located along the vertical Mz (Fig.63). Bearing in mind the diurnal rotation of the earth about the axis of the earth yy, we see that we here have to do with a gyroscope located on a rotating base.

According to Foucault's rule, our gyroscope with two degrees of freedom reacts to the rotation of the earth about its axis, by attempting to assume a corresponding parallelism with the axis of the earth. However, the gyro axis AB cannot become parallel to the axis of the earth, since it must remain parallel to the horizontal plane H.

Under these conditions, the gyro axis tends to approach as closely as possible the direction of the earth's axis and strives to establish itself in the same dir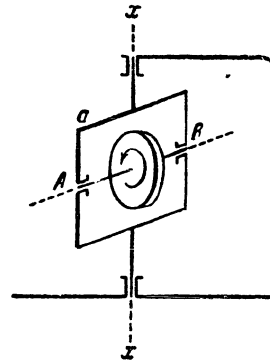ection in the horizontal plane which is closest to the direction of the earth's axis. In the horizontal plane, the direction closest to the direction of the earth's axis is the direction of the meridian ns. For this reason, in reacting to the diurnal rotation of the earth, the gyro axis AB is established in a direction, to be denoted by N_oS_o, parallel to the meridian ns. In this case, the A end of the axis AB will point north and the B end south, so that both rotations, the proper rotation of the gyroscope itself and the diurnal rotation of the earth, are directed in one and the same sense, counterclockwise if viewed from the north.

Thus, in reacting to the diurnal rotation of the earth, our gyro with two de-

Fig.62

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grees of freedom actually does acquire properties analogous to the properties of the magnetic needle: the gyro axis is set along the meridian (of course the geographic, not the magnetic meridian!), one of its ends indicating north and the other south. It follows from this that the gyro with two degrees of freedom, of the design we have just described, actually may be utilized as a mechanical compass.

Our readers must be warned, however, that if any one should wish to verify the above statements by direct experiment with the instrument shown in Fig.62, he would
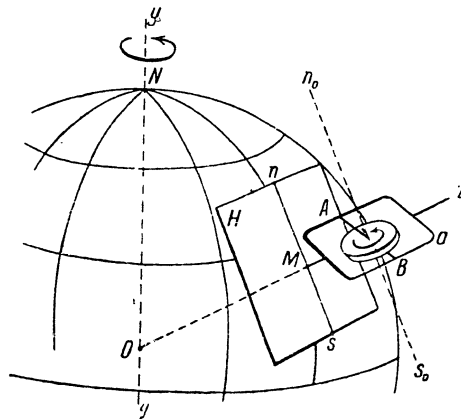

Fig.63

suffer a bitter disappointment. On putting the gyroscope rotor into rapid rotation and placing the instrument on the horizontal plane of a table, the instrument (in contrast to what we have just said) is unable to react to the diurnal rotation of the earth. The gyro axis is not aligned with the direction of the meridian but remains in the same position in which it is placed by the experimenter. The cause for the failure of this experiment is very simple. It is due to the extremely small value of the angular velocity of the diurnal rotation of the earth. For, you see, the

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earth performs only a single diurnal rotation every 24 hours! With such an insignif-
icant value of the angular velocity of the rotation of the base of our instrument,
the forces which would rotate the gyro and align the axis with the direction of the
meridian also remain negligible. These insignificant forces cannot overcome the
resistance of friction at the axis of rotation of the frame A of the instrument.
This is why the gyroscope remains immobile and, apparently, insensitive to the diur-
nal rotation of the earth.

It follows from this that the extreme slowness of the diurnal rotation of the
earth was one of the difficulties that stood in the way of creating the gyro compass.
An instrument sensitive to so insignificant a factor as the angular velocity of the
diurnal rotation of the earth would have to be very perfect, and in particular, the
friction on its axis of rotation would have to be reduced to the lowest possible
minimum. The level at which technology stood in the time of Foucault made the reali-
zation of his idea impossible at the time. The subsequent progress of technology has
removed many of the difficulties that previously were insurmountable; however, there
was still another and more serious difficulty which confronted the designer of the
mechanical compass.

When we speak of a mechanical compass, we primarily have in mind a seagoing com-
pass, i.e., a compass that must operate on shipboard. The floating ship, of course,
participates in the diurnal rotation of the earth, but in addition to this it also
performs various other rotary motions. In its maneuvers, the ship makes turns to one
side or the other; its rolling with the waves is likewise accompanied by the gener-
ation of various rotary motions of the ship's hull. In this case, the angular veloc-
ities of these rotary motions (turns, rolls,) are many times as great as the insignif-
icant value of the angular velocity of the diurnal rotation of the earth.

In order to operate properly, a mechanical compass on a ship must react to the
insignificant angular velocity of the diurnal rotation of the earth, and at the same
time must be insensitive to the far more considerable angular velocities of the other

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rotary motions of the ship. How can such an instrument be built? At first glance the problem seems insoluble. In the following Sections we will discuss the manner in which the problem was solved in modern designs of the gyro compass.

Section 22. Rotation of the Plane of the Horizon About the Vertical and

About the Meridian.

Let us first discuss one question which we mentioned briefly in the preceding pages, namely, that of the rotation of the plane of the horizon about the vertical and about the meridian. This phenomenon is a direct consequence of the diurnal rotation of the earth.
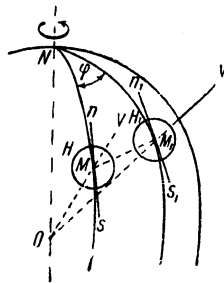

Fig.64

Let us imagine the earth again, and let us take the point M on the earth's surface (Fig.64). At this point, we mark the vertical MV and the horizontal plane H, on which latter we mark the meridian ns. As a result of the diurnal rotation of the earth, each terrestrial meridian rotates through an angle of $\frac{360^{\circ}}{24} = 15^{\circ}$ every hour about the axis of the earth; each point of the earth's surface is displaced along the arc of a parallel of latitude. During the small time interval τ, the meridian at the point M rotates through the small angle φ, the point M is displaced to the position M_1, while the vertical MV and the plane of the horizon H are displaced, together with the point M, to the positions $M_1 V_1$ and H_1, respectively.

The displacement of the plane of the horizon H_1 can be imagined as consisting of two components: 1.)the displacement of the plane H together with the point M, not accompanied by its rotation, and 2)the rotation of the plane H about the point M.

The first component of the displacement of the plane of the horizon is of no interest

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to us, since the gyro placed on the horizontal plane H in no way reacts to a dis-
placement of the plane of the horizon, not accompanied by a rotation; it is sensi-
tive only to the rotary motion of the base.

This gives us the right, in our subsequent discussion, to disregard the displace-
ment of the point M and to limit ourselves to considering the rotation of the plane
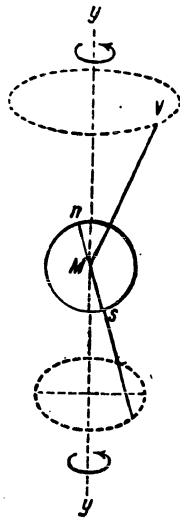of the horizon about the point M. Accordingly, let us draw through the point M a
straight line yy parallel to the H axis, and
let us consider that the rotation of the
plane of the horizon H, the vertical M, and
the meridian ns takes place about the yy ax-
is (Fig.65).

Let us follow the rotation of the verti-
cal MV and the meridian ns during the time
γ . During this time, the earth rotates
about the earth's axis through the angle ;
the vertical MV and the meridian ns also ro-
tate through the same angle about the axis yy.
From the point M in the direction of the
vertical MV and of the meridian ns, lay off
equal segments MA = MB (Fig.65), and from the
points A and B drop the perpendiculars AC
and BD to the axis yy. In the time γ , the
segments MA and MB, rotating about the axis

Fig.65

yy through the angle φ , will occupy the positions MA_1 and MB_1, while the triangles
MAC and MBD will occupy the positions MA_1C and MB_1D. Let us denote the angle BMB_1
by φ_1 and the angle AMA_1 by φ_2.

The displacement of the segments MA and MB to the positions MA_1 and MB_1 (and at
the same time the displacement of the horizontal plane H from its first position to

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its second position) is accomplished by a rotation about the axis yy through the angle φ . This same displacement can proceed by two successive rotations about the vertical and about the meridian. Let us turn the plane AMB about the line MA through the angle φ_1; after this rotation it will occupy the position AMB_1. Now let u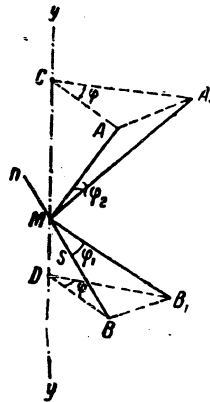s rotate it about the line MB_1 through the angle φ_2, after which the two segments MA and MB will, in their new positions, be MA_1 and MB_1. In this way, the rotation of the segment MA and MB about the yy axis through the angle φ is equivalent to two successive rotations about the vertical and about the meridian through the angles φ_1 and φ_2, respectively.

These two rotations are termed component rotations.

We have now resolved the diurnal rotation about the axis of the earth into two component rotations, one about the vertical and one about the meridian.

Fig.66

We note (this will be clear from Fig.66) that to the observer looking from the northern end of the meridian, the rotation about it appears counterclockwise. As a result of this rotation of the horizon about the meridian, the heavenly bodies appear to ascend in the celestial vault in the eastern half of the sky and to sink toward the horizon in the western half.

It is also easy to see from Fig.66 that the rotation of the horizon about the vertical will be counterclockwise (i.e., from right to left) for an observer looking from above, i.e., from that part of the vertical which is turned toward the zenith.

For this reason, the heavenly bodies appear to us to be displaced clockwise along

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the celestial vault in its diurnal motion (i.e., from left to right*).

Let us denote the angular velocity of the diurnal rotation of the earth by the symbol Ω and the angular velocities of the components of the rotation about the vertical and about the meridian by Ω_1 and Ω_2, respectively. To find the relationship between the angular velocities, let us now turn again to Fig.66.

Draw the segments AA_1 and BB_1 and note that, in view of the smallness of the angle φ, these segments may be considered equal to the arcs of the circumferences with centers C, M, and D and with radii CA, MA, MB, and DB. Since an arc is equal to the product of the radius and the corresponding central angle, we may write:

$$AA_1 = CA \cdot \varphi = MA \cdot \varphi_2, \quad BB_1 = DB \cdot \varphi = MB \cdot \varphi_1$$

whence

$$\varphi_1 = \frac{DB}{MB} \varphi , \qquad \varphi_2 = \frac{CA}{MA} \varphi$$

On dividing these equations by τ, and noting that

$$\Omega = \frac{\varphi}{\tau}, \qquad \Omega_1 = \frac{\varphi_1}{\tau}, \qquad \Omega_2 = \frac{\varphi_2}{\tau}$$

we obtain

$$\Omega_1 = \frac{DB}{MB} \Omega , \qquad \Omega_2 = \frac{CA}{MA} \Omega$$

---

* The reservation must be made that what has just been said about the direction of the rotation of the horizon about the vertical relates only to the northern hemisphere. In Fig.64 we have assumed the point M to lie in the northern hemisphere. If we take the point M in the southern hemisphere and repeat all our constructions, then it is easy to convince ourselves that in the southern hemisphere the rotation of the horizon about the vertical is clockwise for an observer looking from above. At the equator, there is no rotation of the horizon about the vertical at all.

It is clear from Fig.66 that the segments CA and DB are less than the segments MA = MB = a. Consequently, the quantities $\frac{CA}{MA}$ and $\frac{DB}{MB}$ are proper fractions. We conclude that the angular velocities Ω_1 and Ω_2 are less than the angular velocity of the diurnal rotation of the earth.

Let us denote the latitude of the point M on the earth's surface by the symbol (Fig.67). The angle formed by the direction of the earth's radius OM or by
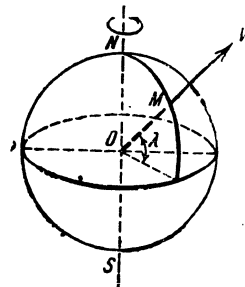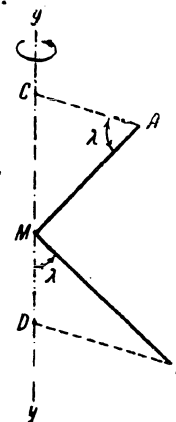

Fig.67

Fig.68

the direction of the vertical MV with the earth's axis SN is equal to $90^\circ - \lambda$.

Let us reproduce the triangles MAC and MBD (cf. Fig.66) in Fig.68. Since the direction of the segment MA coincides with the direction of the vertical MV, it follows that $\angle AMC = 90^\circ - \lambda$ and that, consequently, $\angle CAM = \lambda$. The sides of the angle DMB are perpendicular to the sides of the angle CAM; consequently, the angle BMD also equals λ .

Now, from the right triangles MAC and MBD, we have:

$$CA = MA \cos \lambda \; , \; DB = MB \sin \lambda$$

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whence

$$\frac{CA}{MA} = \cos \lambda \ , \ \frac{DB}{MB} = \sin \lambda$$

By substituting these values of the quantities $\frac{CA}{MA}$ and $\frac{DB}{MB}$ in the formulas for Ω_1 and Ω_2, we find that

$$\Omega_1 = \Omega \sin \lambda \ , \ \Omega_2 = \Omega \cos \lambda$$

In this way, the angular velocities of rotation of the plane of the horizon about the vertical and about the meridian are expressed in terms of the angular velocity of the diurnal rotation of the earth and in terms of the latitude of the place.

These formulas yield, for Leningrad: $\Omega_1 = \sin 60^\circ = 0.866 \ \Omega$ and $\Omega_2 = \cos 60^\circ = 0.5 \ \Omega$, which corresponds to a rotation of the horizon about the vertical and about the meridian through 312° and 180° respectively, every 24 hours.

Section 23.   The Sperry Gyro Compass with Pendulum

We already know that, theoretically speaking, a gyro with two degrees of freedom can be used as a mechanical indicator of the direction of the meridian, i.e., as a mechanical compass. However, the task of designing such an instrument proved to be on solid ground only when the designers of the instrument abandoned the thought of using a gyro with two degrees of freedom and used a gyro with three degrees of freedom instead.

At first glance this seems surprising, since it is known that the astatic gyro with three degrees of freedom is absolutely insensitive to the rotation of the base. How could it thus be possible to use such a gyro in an instrument which, by its very purpose, must react to the diurnal rotation of the earth? Of course, an astatic gyro with three degrees of freedom is entirely free of any directional force turning the gyro axis toward the meridian. To produce such a force, a special device would be necessary to guarantee the generation of the directional force and thus to convert the gyro into a gyro compass.

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In the gyro compass of the famous American builder of gyroscopic instruments, Sperry, who in 1911 built a marine instrument which proved to be entirely suitable for practical purposes, the idea of combining a gyro with an ordinary pendulum was used. Let us consider the scheme of this instrument.

The basic element of the instrument is a gyro of the type discussed above, in a Cardanic suspension, with a vertically located axis of rotation of the outer ring R (Fig.69). The inner ring S is rigidly 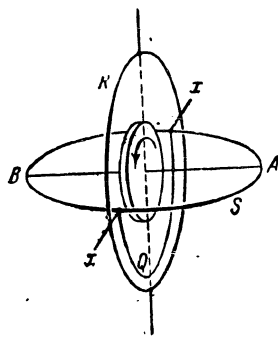connected to the heavy arc Q, whose plane is perpendicular to the plane of the inner ring. The arc Q, at the same time, represents the pendulum which in this instrument is combined with the gyro. This pendulum is able to swing (together with the inner ring S and the gyro rotor) about the axis of rotation xx of the inner ring. It is found that such a simple instrument is able to function as a gyro compass since the pendulum Q supplies its directional force.

Now let us consider the behavior of the instrument, assuming that the gyro rotor is

Fig.69

rotating about the axis AB, e.g., counterclockwise for an observer viewing it from the A end of the axis AB (as shown by the curved arrow in Fig.69). Let us at first disregard the diurnal rotation of the earth, i.e., let us assume that the instrument is located on a fixed base.

If the axis AB of the rotor is directed horizontally, the pendulum Q will be vertical (Fig.69) and in its equilibrium position, with the entire instrument remaining constant in space. If the rotor axis AB is moved from the horizontal position by raising its A end and by lowering its B end (Fig.70), then the pendulum Q will also be moved from its vertical equilibrium position. Under the action of its own

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weight, it will tend to return to the vertical position, but to do this will disturb its connection with the gyro. At the points A and B, the system of the pendulum and the inner ring will exert pressure on the rotor axis of the gyro by the forces F_1 and F_2, tending to lower the A end and raise the B end of this axis (Fig.70). What will be the result of the action of the forces F_1 and F_2 on the motion of the gyro axis? Remembering the rule of precession (Section 6) and also what has been said in
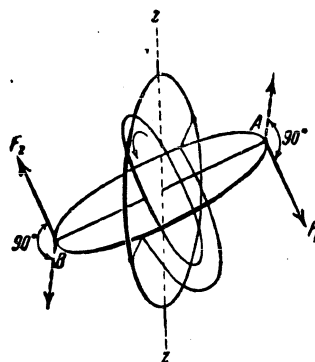
Section 10 on the precession of the gyro due to the application of continuously acting forces to its axis, we conclude that the gyro axis will precess, rotating about the vertical axis of rotation zz of the outer ring. According to the rule of precession, let us rotate the directions of the forces R_1 and R_2 about the axis AB through 90° in the sense of rotation of the rotor; this will make it possible to find

Fig.70

the direction of motion of the points A and B. We will see that the precessional rotation of the gyro axis about the axis of rotation zz of the outer ring of the suspension will appear counterclockwise to an observer viewing the instrument from above.

Thus, under the action of its own weight, the pendulum does not descend, but instead the whole instrument will rotate counterclockwise about the vertical. On the other hand, if we lower the A end of the rotor axis and raise the B end, then the action of the force of gravity will cause the pendulum of the instrument to rotate clockwise about the vertical.

Such is the behavior of the gyro with a pendulum on a fixed base. Let us now

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take into account the diurnal rotation of the earth and let us see how our instru-
ment will react to the diurnal rotation of the earth.

Let us assume that, at a certain point of the earth's surface (in the northern
hemisphere), our instrument is placed in a horizontal plane and that the gyro axis
is brought into a horizontal position. We assume further that the gyro axis AB
forms a certain angle α with the meridian ns, in such a way that the A end of the
axis AB is east of the meridian line while the B end is west of it (Fig.71; this
drawing shows neither the rings of
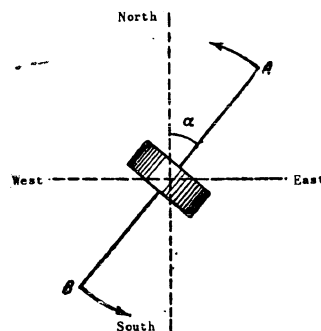the gyro suspension nor the pendu-
lum).

So long as the gyro AB remains
horizontal and, consequently, the
pendulum is vertical, the instru-
ment has no tendency to vary its
position in space. If the end A of
the gyro axis is pointed at some
star, it will stably maintain its
direction toward that star. How-
ever, as a result of the diurnal ro-
tation of the earth, the star to-
ward which the A end of the gyro axis is pointed (if it is on the horizon in the
eastern half of the celestial vault) will gradually begin to rise above the horizon.
At the same time, the A end of the gyro axis will also gradually rise above the
plane of the horizon, and the opposite B end of the gyro axis will begin to sink be-
low the plane of the horizon. In this way, as a result of the rotation of the earth,
the gyro axis gradually leaves its horizontal position by raising the A end and low-
ering the B end. At the same time, the pendulum also leaves its vertical position;
we have just seen that under the action of its force of gravity, the entire instru-

Fig.71

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ment in this case begins to rotate counterclockwise about the vertical (as shown by the curved arrows in Fig.71), i.e., in the direction of decreasing angles or in the direction of approach of the gyro axis AB to the meridian ns. It is easy to show, by a similar argument, that the reverse rotation of the gyro axis about the vertical will take place as a result of the rotation of the earth in the case where the gyro axis AB deviates with its A end to the west of the meridian and with its B end to the east of it (Fig.72); in this case, the rotation of the earth communicates to the instrument a tendency to rotate about the vertical, attempting to bring the
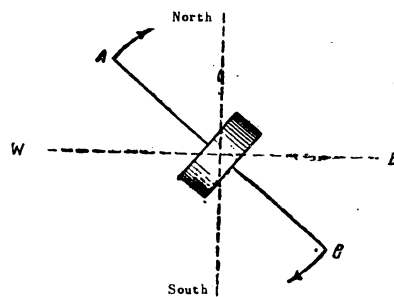

Fig.72

gyro axis AB closer to the meridian ns.

Thus, in all cases when the gyro axis AB deviates from the meridian, the instrument, reacting to the diurnal rotation of the earth, will acquire a tendency to rotate about the vertical and to place the axis AB along the meridian in such a way that the A end of this axis points north and the opposite end points south (we recall that we used the letter A to denote that end of the gyroscope axis from which the rotation of the rotor appeared counterclockwise, Fig.69).

As we see, the gyro with a pendulum of the above design actually can serve as a mechanical indicator of the meridian. Under the influence of the diurnal rotation

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of the earth, the axis of such a gyro acquires properties analogous to the properties of the magnetic needle (with the difference, however, that the magnetic needle gives the direction of the magnetic meridian, while the axis of our gyro is set along the geographic or true meridian). The combination of the above type of pendulum with a gyro having three degrees of freedom imparts to this gyro, under the influence of the earth's rotation, a directional force and transforms it into a gyro compass. The A end of the gyroscope axis AB, viewed from the point from which the proper rotation of the rotor appears to be counterclockwise, may be termed the north end of the gyro compass axis, and the B end its south end. This is the principle of operation of the Sperry gyro compass with pendulum.

Section 24. Inclination of the Axis of a Gyro Compass in its Equilibrium Position.

We have just seen that, on deviation of the axis of the gyro compass from the meridian line, the influence of the earth's rotation generates a directional force tending to return that axis to the direction of the meridian. Let us now consider more closely the equilibrium position occupied by the gyro compass when its axis is in fact directed along the meridian.

It is erroneous to believe that, once the axis of the gyro compass is aligned horizontally with the direction of the meridian, it will remain invariantly directed along the meridian. It must be taken into account that, as a result of the earth's rotation, the meridian itself will rotate, together with the plane of the horizon, about the vertical (counterclockwise in the northern hemisphere, clockwise in the southern) with an angular velocity which we have denoted in Section 22 by $1^\bullet$. Consequently, in order to have the gyro-compass axis remain invariantly directed along the meridian, the entire instrument will have to rotate counterclockwise about the vertical (if we are in the northern hemisphere), with the same angular velocity $1^\bullet$. On the other hand, we already know (cf. Section 23) that our gyro acquires a precessional rotation about the vertical, counterclockwise if the gyroscope axis

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is raised by its north end above the plane of the horizon (and clockwise if it is depressed below that plane). It follows from this that, in the northern hemisphere, the axis of the gyro compass will invariantly maintain the direction of the meridian if its north end is raised by a certain angle (which we shall term β_o) over the plane of the horizon. If the north end of the gyro-compass axis is raised above the plane of the horizon by an angle greater than β_o, then the axis of the gyro compass will rotate counterclockwise about the vertical, with an angular velocity greater than Ω_1; consequently, its north end will gradually depart from the meridian, deviating from it to the west. On the other hand, when the north end of the gyro compass is elevated above the plane of the horizon by an angle less than β_o, then the gyro-compass axis will depart in its rotation about the vertical from the meridian, and its north end will depart from the meridian and will deviate from it to the east.

Thus, the axis of the gyro compass keeps the direction of the meridian constant in the case in which it is directed in the plane of the meridian, and if its north end is raised (in the northern hemisphere) above the plane of the horizon by the angle β_o. In the southern hemisphere, the north end of the gyro-compass axis, in its equilibrium position, is depressed by the corresponding angle below the plane of the horizon. It is only on the equator that the axis of the gyro compass assumes a constant direction along the meridian when the axis of the instrument is in the horizontal position.

As for the magnitude of the angle of tilt of the axis of the gyro compass in its equilibrium position, i.e., the angle β_o, this angle is very small. The value of the angle β_o depends on the latitude of the place; we have seen that, on the equator, the angle β_o is equal to zero. A more detailed calculation whose details will not be given here, shows that at the latitude of Leningrad (i.e. at Lat. $\lambda = 60°$) the angle β_o in the Sperry gyro compass is equal to a few minutes of arc (about 8 minutes or $\frac{8}{60}$ degrees).

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Section 25. Undamped Oscillations of the Axis of the Sperry Gyro Compass.

We have just seen that the gyro-compass axis remains aligned with the meridian only in the case where it is given a corresponding inclination, by the angle β_o, to the plane of the horizon. How will the axis of a gyro compass behave if it is directed in the plane of the meridian but is not given the angle of inclination β_o corresponding to the latitude of the place? Let us further consider this question.

Let us assume that the axis of the gyro compass lies in the plane of the meridian, but without the necessary inclination (by the angle β_o) to the plane of the
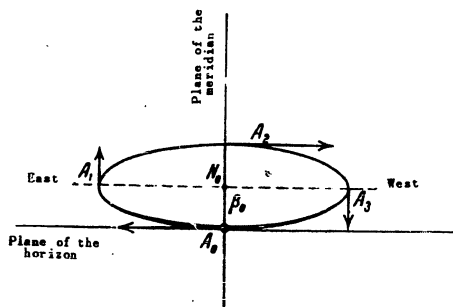

Fig.73

horizon. Let us follow up the subsequent motion of the north end of the gyro-compass axis, viewing the instrument from the north, i.e., turning our face to the south. In this case, the east will be on our left side and the west on our right (Fig.73).

As already stated, the north end of the gyro-compass axis lies in the plane of the horizon to the north. In Fig.73, which shows the displacement of the north end of the gyro-compass axis as it appears to an observer viewing it from the north side, i.e., turning his face to the south, the north end of the gyro-compass axis (denoted, for brevity, by the letter A) is at first in the position A_o (at the intersection of the plane of the horizon and the plane of the meridian). In this same Fig.73, the

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equilibrium position N_0 of the north end of the gyro-compass axis is also shown, namely the plane in which it is raised above the plane of the horizon by the angle β_0.

At first, when the north end A of the gyro-compass axis is in the position A_0, the pendulum is vertical, and the gyro-compass axis has no tendency to vary its position in space. As a result of the rotation of the plane of the horizon about the vertical with the angular velocity Ω_1, however, this plane, together with the meridian, will deviate from the north end of the gyro-compass axis toward the west[*]. Instead of this it appears to us that the north end A of the gyro-compass axis deviates from the meridian toward the east (Fig.73). However, we already know (cf. Section 23, Fig.71) that, when the north end of the gyro-compass axis deviates from the meridian toward the east, it begins to rise above the plane of the horizon (more exactly, the plane of the horizon begins to sink beneath it as a result of the rotation of the plane of the horizon about the meridian). On following the motion of the north end A of the gyro-compass axis, we see that the point A not only departs from the meridian toward the east but, at the same time, rises above the plane of the horizon (Fig.73).

When the gyro-compass axis ceases to point in a horizontal direction, the pendulum also deflects from its vertical position and begins to act on the gyro rotor. We already know that this causes a precessional motion of the gyro-compass axis in the direction of the plane of the meridian (from east to west) which, as the angle of inclination of the pendulum from the vertical increases, will gradually be accelerated. As a result of this, the deviation of the axis of the gyro compass from the plane of the meridian gradually becomes less but does not stop at the moment when the angle of inclination of the gyro-compass axis reaches the value β_0, corresponding to the equilibrium position of the gyro compass. At this moment, the point A

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[*] We recall (Section 22) that the rotation of the horizon about the vertical appears to be counterclockwise in the northern hemisphere.

reaches the position A_1 (Fig.73), decribing the arc A_0A_1.

However, the A end of the gyro-compass axis is unable to stop in the position A_1, since its equilibrium position is the position N_0 in the plane of the meridian. Be-ing to the east of the meridian, the point A continues to rise above the plane of the horizon; the precessional motion of the gyro-compass axis from east to west con-tinues to be accelerated, and the apparent deviation of the gyro-compass axis from the plane of the meridian is now replaced by an apparent motion opposite to it in the plane of the meridian. The point A continues its motion, rising above the plane of the horizon and at the same time returning to the meridian; it describes the arc A_1A_2 (Fig.73).

In the position A_2, the point A again is in the plane of the meridian, but this time the gyro-compass axis is raised above the plane of the horizon, not by the an-gle β_0, but by a larger angle (more specifically, by the angle 2 β_0). As a re-sult of this, the A end of the gyro-compass axis, on passing through the position A_2, begins to deviate to the west of the plane of the meridian, and the north end A of the gyro-compass axis gradually begins to descend toward the horizon (like a star in the western part of the heavenly vault). At the same time, the deviation of the gyro-compass axis from the plane of the meridian gradually lessens and finally stops at the instant when the angle of inclination of the gyro-compass axis again reaches the value β_0, corresponding to the equilibrium position of the gyro compass. At this instant, point A, describing the arc A_2A_3, reaches the position A_3 (Fig.73).

Subsequently, the northern end A of the gyro-compass axis, being to the west of the meridian, continues to descend, and its visible deviation from the plane of the meridian is replaced by an apparent motion in reverse, toward the plane of the merid-ian. The whole cycle of motions of the point A is accomplished at the instant when this point, describing the arc A_3A_0, again must pass through its initial position A_0, after which the entire cycle, with relative to the motions of the point A is repeat-ed anew an indefinite number of times.

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We reach the conclusion that if the gyro-compass axis is placed in the plane of
the meridian but is not given the corresponding equilibrium position of inclination
(by the angle β_o) to the plane of the horizon, the gyro-compass axis will not
maintain its position in the plane of the meridian; it will begin to fluctuate about
its own equilibrium position N_o, executing oscillations in both horizontal and ver-
tical directions, the horizontal between the positions A_1 and A_3 and the vertical
between the position A_o and A_2. The north end A of the gyro-compass axis describes
the oval curve $A_oA_1A_2A_3A_o$, which is termed an ellipse.

In this way, if the gyro-compass axis is not in its equilibrium position N_o, it
is unavoidable that the influence of the diurnal rotation of the earth will cause
this axis to execute elliptical fluctuations about this position. In view of the
extreme smallness of the angle β_o, the elliptical path $A_oA_1A_2A_3A_o$ of the end of
the gyro-compass axis appears very much elongated in a horizontal position. For
this reason, the oscillations of the gyro-compass axis in the fundamental direction
are, on the whole, directed in the horizontal plane and are accompanied by only
insignificant vertical fluctuations. These fluctuations are not damped in the
course of time and, therefore, are termed undamped oscillations of the gyro-compass
axis.

The time of one complete oscillation, i.e., the time in which the north end A of
the axis of the gyro compass describes its elliptical path $A_oA_1A_2A_3A_o$ is called the
period of undamped oscillations of the gyro-compass axis. This time depends on the
dimensions and weights of the parts of the gyro compass, on the angular velocity of
the proper rotation of the gyro-compass rotor, and also on the angular velocity of
the diurnal rotation of the earth and on the latitude of the place in question. The
considerations given below, force us to select the design elements of the gyro com-
pass in such a way that the period of its undamped oscillations is equal to approxi-
mately 84 minutes (a Schuler period).

The period of the undamped oscillations of the gyro-compass axis with pendulum

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(denoted by the letter T) is determined by the formula:

$$T = 2\pi \sqrt{\frac{J\,w}{pa\,\Omega\,\cos\lambda}}$$

where J is the moment of inertia of the gyro-compass rotor, p the weight of the pendulum, a the distance of its center of gravity from the horizontal axis of rotation xx of the inner Cardanic ring (Fig.69), w the angular velocity of the proper rotation of the rotor, Ω the angular velocity of the diurnal rotation of the earth, and λ the latitude of the place. As will be clear from this formula, the more rapidly the rotor of the gyro compass is rotating, the more slowly will its undamped oscillations take place.

Section 26. Eccentric Coupling of the Pendulum with the Gyro Chamber.

We have just seen that continuous oscillations of the axis of the gyro compass about its equilibrium position are an unavoidable accompaniment of the operation of a Sperry-type gyro compass with a pendulum of the above-described design. Merely by placing the axis of the gyro compass in the equilibrium position, i.e., by directing it in the plane of the meridian and giving it the necessary inclination to the plane of the horizon (by the angle denoted by β_0) the generation of such fluctuations can be avoided. The Sperry gyro compasses contained a special device to impart to the gyro-compass axis the necessary inclination to the plane of the horizon (for so-called leveling of the axis). Such measures, however, are insufficient to completely avoid the generation of undamped oscillations of the gyro-compass axis, which substantially hamper its operation. To make the Sperry gyro compass useful in practice, its design, as mentioned in Section 23, had to include a slight modification to ensure the necessary extinction of the generated oscillations of the gyro-compass axis. This modification is very remarkable; it shows that in gyroscopic systems we may still attain extinction of oscillations without introducing any resistances to motion into the system (forces of friction, etc.), which always

have a harmful effect on the operation of the instrument.

We note first of all that the principal scheme of the instrument described in Section 23, remains unchanged if we do not couple the pendulum rigidly to the inner gimbal ring in such a way that this ring and the pendulum constitute a single whole (as was assumed in Section 23 and as was depicted in Fig. 69), but instead, assume the pendulum to be suspended in the following manner:

In the Sperry gyro compass, the outer gimbal ring (1) (Fig.74) is surrounded by another ring (2). This ring is also able to rotate (like ring (1)) about the vertical axis (3-3) and is rotated by a special motor (which is called the "azimuth motor"), which automatically holds the ring (2) in the same plane as the outer gimbal ring; in this way the ring (2),as it were, follows the displacements of the outer gimbal ring (1) and for this reason is termed the "follow-up" ring. It is in order to follow this ring (2) that the pendulum (4) is suspended on the horizontal axis (5-5).
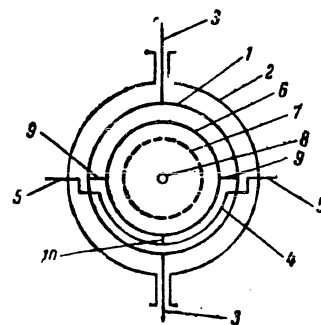
Fig.74

The role of the inner gimbal ring in the Sperry gyro compass is played by the round chamber (6) in which the gyro rotor (7) is placed and to which the axis of rotation of the rotor (8) is attached. The horizontal axis of rotation of the chamber (9-9) is attached to the outer gimbal ring (1). The coupling of the gyroscopic system to the pendulum is accomplished by means of the pin (10) which couples the pendulum (4) to the chamber (6).

If this pin (10) is placed at the lowest point of the chamber (6), the general

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layout of the instrument in question does not differ basically from the scheme dis-
cussed in Section 23, and as there, it is obvious that the inclination of the gyro-com-
pass axis from the plane of the horizon, accompanied by the corresponding deviations
of the pendulum from the vertical position, causes a precessional motion of the in-
strument about the vertical; this precessional rotation is counterclockwise if the
north end of the gyro-compass axis is raised above the plane of the horizon, and is
clockwise if this end of the axis is depressed below that plane. The reason for
this precessional rotation is the pressure of the pendulum, transmitted to the rotor

axis through the medium of the pin
(10) and of the gyro chamber (6).

It is easy to see that all fur-
ther deductions made in Sections 24
and 25 remain in force without any
modification whatever. Of course,
we do assume (as in Section 23)
that the gyro rotor is rotating
counterclockwise if viewed from the
north.

The behavior of our instrument
will change substantially if we as-
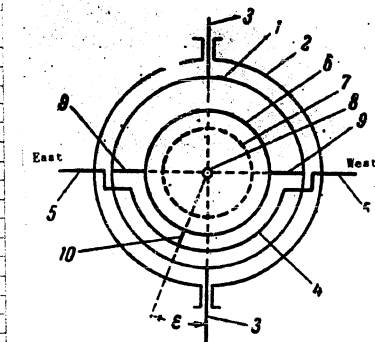
Fig.75

sume that the gyro chamber is coupled with the pendulum by means of the pin, not at
the lowest point of the chamber but somewhat displaced from this point to the east
(Fig.75). This was exactly what was done in the Sperry gyro compass. This simple
and remarkable modification of the above-mentioned design of the gyro compass made
it possible to ensure damping of the oscillations of the gyro-compass axis, a modi-
fication commonly known as "eccentric coupling" of the compass and the gyro chamber.

Let us describe the functioning of an instrument of this design. First, we dis-
regard the effect of the diurnal rotation of the earth. Let us imagine that the

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north end of the gyro-compass axis ((8) in Fig.76) is raised above the plane of the

horizon (and that, consequently, the pendulum (4) is inclined by the same angle from

the vertical plane toward the north, Fig.76). Then, the pressure S transmitted from

the pendulum (4) over the pin (10) to the gyro chamber (6)[*], and over that chamber

to the rotor axis, will tend to rotate the rotor axis toward the horizontal position
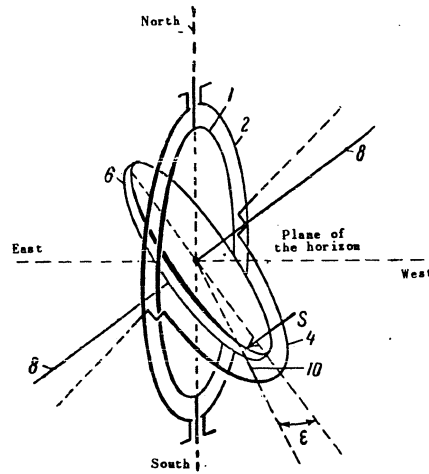
and at the same time tend to turn the gyro chamber and the rotor axis about the

Fig.76

vertical axis (3-3) in a clockwise direction (in Fig.76, the gyro chamber is shown

schematically in the form of a flat or round disk).

Consequently, the forces F_1 are transmitted through the chamber to the rotor ax-

is. These forces lie in the vertical plane containing the axis (3-3) and the hori-

zontal forces F_2 (Fig.77). We recall the rule of precession (Section 6). By

* Note that the pressure is directed parallel to the gyro-compass axis (8).

rotating the forces F_1 and F_2 about the rotor axis through $90°$ in the sense of the rotation of the rotor (the sense of rotation of the rotor is indicated in Fig.77 by the curved arrow), the action of the force F_1 causes the gyro axis to precess, rotating counterclockwise about the vertical axis (3-3); at the same time, the action of the forces F_2 causes the north end (i.e., the right-hand end in Fig.77) of the gyro axis to descend toward the horizon, and the southern end to rise.
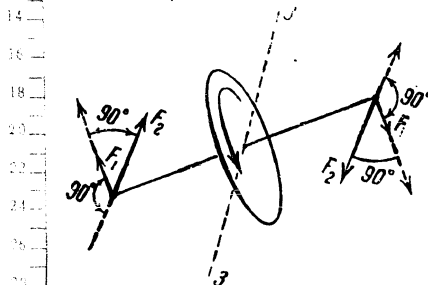
Thus the variation in the inclination of the gyro axis to the horizon is due to this new effect which is introduced by the "eccentric coupling" of the pendulum with the gyro chamber in the operation of the instrument when the gyro

Fig.77

axis deviates from the plane of the horizon. This effect was absent when the pendulum was rigidly coupled with the inner ring of the gyro or with the gyro chamber (Section 23) and it must also be absent when the pendulum is coupled with the gyro chamber by means of a pin fitted at its lowest point, i.e., with $\eta = 0$ (Fig.74), since in this case the forces F_2 are absent. We shall see later how this effect, produced by the "eccentric coupling" of the pendulum with the gyro chamber, is utilized to damp the oscillations of the gyro compass.

Section 27. Deviations of the Gyro Compass Due to the Eccentric Coupling of the
                   Pendulum with the Gyro Chamber, or Fluctuations in Damping.

Before discussing the mechanism of the damping of oscillations in a gyro compass, due to the eccentric coupling of the pendulum with the gyro chamber, let us mention

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still another peculiarity introduced into the operation of the instrument by this eccentric coupling. We already know that, in the equilibrium position, the axis of the gyro compass is located in the plane of the meridian and receives a certain inclination to the horizon; the north end of the gyro-compass axis is raised (in the northern hemisphere) above the plane of the horizon by an angle which we have denoted in Section 24 by β_0. The inclination of the gyro-compass axis by this angle ensures the precessional rotation of its axis about the vertical (counterclockwise) with an angular velocity Ω_1 of the rotation of the plane of the horizon about the vertical.

As a result of this, the gyro-compass axis follows the rotation of the meridian about the vertical (due to the diurnal rotation of the earth) and invariantly remains in the plane of the meridian.

This is the situation in the absence of "eccentricity" in the coupling between the pendulum and the gyro chamber i.e., at $\epsilon = 0$ (Fig.74). How does the existence of such eccentricity, i.e., the placing of the pin coupling the compass with the gyro chamber at the angular distance   toward the east, affect the equilibrium position of the gyro-compass axis?

We know that, in this case, the rise of the north end of the gyro-compass axis produces not only the precessional rotation of the axis of the instrument about the vertical, but also a gradual lowering of this axis toward the plane of the horizon. To ensure a constant position of the gyro-compass axis with respect to the surrounding terrestrial objects in the equilibrium position of the instrument, it is necessary in some way to eliminate this gradual descent of the north end of the axis of the instrument in the equilibrium position. How can this be done?

We know already that, in consequence of the diurnal rotation of the earth, every easterly deviation of the north end of the gyro-compass axis imparts to it a tendency to a gradual rise above the level of the horizon (we recall the star rising on the eastern side of the sky as a result of the diurnal rotation of the earth). Let

us tilt the north end of the axis of our gyro compass from the plane of the meridian toward the east by some angle β_o, and let us select this angle in such a way that the resultant easterly deviation will make it possible for the gradual rise in the axis of the instrument to compensate the lowering of the axis due to the "eccentric coupling" between the pendulum and the gyro chamber. We conclude that, in the presence of an eccentric coupling, the constant position of the axis of the instrument with respect to the surrounding ground objects will be obtained in the case when the north end of the instrument axis in this equilibrium position is raised above the plane of the horizon by the angle β_o and is simultaneously caused to deviate from the plane of the meridian to the east by the angle β_o. A more detailed investigation of the question allows us to determine the value of the angle β_o. It develops that this angle depends on the size of the angle ε (i.e., on the value of the angular easterly deviation of the pin in the "eccentric coupling") and on the latitude of the place. In the Sperry gyro compass, ε is taken as 2°, at the latitude of Leningrad, the value for this gyro compass is $\alpha = 3.5^\circ$.

Thus, in the presence of an "eccentric coupling" of the pendulum with the gyro chamber, the axis of the chamber will reach its equilibrium position not along the direction of the meridian, but at a certain angle α_o to the direction of the meridian. This angle α_o is termed the deviation of the gyro compass due to "eccentric coupling" or "deviation of damping".

In this way, the introduction of "eccentric coupling" in the gyro compass causes a substantial error in the instrument readings. Should we not conclude from this that the idea of "eccentric coupling" is entirely unsuitable? By no means. It must be remembered that here we are speaking of a gyro compass installed on a ship and serving to determine the course of the ship, i.e., the angle formed between the direction of motion of the ship and the direction of the meridian. The direction of motion of the ship, in calm weather and in quiet water, coincides with the direction of the ship from stern to prow; this direction, the course line, is given by the

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"course" mark (1) of Fig.78, placed on the round course ring (2) which surrounds the

horizontal card (3), graduated in degrees and rigidly coupled with the follow-up

ring of the gyro compass. If there were no deviation due to "eccentric coupling",

the axis of the gyro compass and, consequently, also the zero division of the card,

would give the direction of the true meridian, while the reading taken from the card

opposite of the course line would directly determine the course of the ship. How-

ever, we know that the "eccentric coupling" causes a certain deviation (easterly in
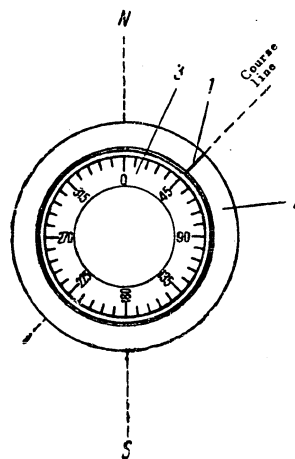
Fig.78
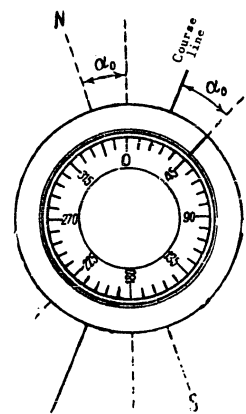
Fig.79

northern latitudes) of the instrument axis and, consequently, also of the zero read-

ings connected with the follow-up ring of the card, by the angle α_0. This intro-

duces an error into the course reading of the gyro compass. It is easy to show,

however, that this error will be compensated by rotating the course ring with the

course mark toward the east through the same angle α_0 (Fig.79). Now the reading

taken on the card will again give the ship's true course.

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In the Sperry gyro compass, a simple device was provided which permits without any preliminary calculations a rotation of the course ring through the necessary angle (the so-called "mechanical correcting device"), in this way compensating the error in the instrument readings due to the deviation caused by the "eccentric coupling".

Section 28. Damping of the Oscillations of the Gyro Compass Axis, Due to Eccentric Coupling of the Pendulum with the Gyro Chamber.

Below, we will discuss the process of damping the oscillations of the gyro-compass axis by using an "eccentric coupling" of a pendulum with a gyro chamber.
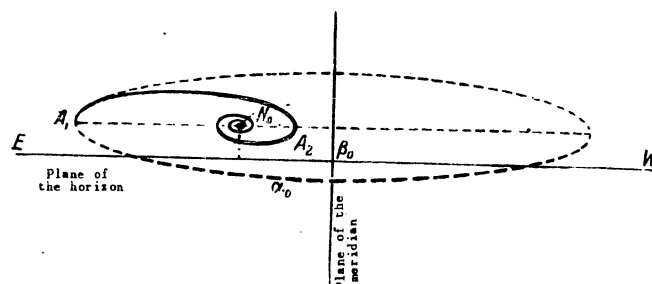
Fig. 80

Let us imagine that we are viewing the instrument while standing to the north of it and facing south (as in Section 25). We mark the position of the north end of the gyro-compass axis on the drawing (Fig. 80), on which the plane of the meridian and the plane of the horizon are shown; the east will be to our left and the west to our right. Let us mark, on this drawing, the equilibrium position N_0 of the north end of the gyro-compass axis; in this position, it is raised above the level of the horizon by the angle α_0 and deviates from the plane of the meridian to the east by

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the angle θ_0.

Let us assume that the axis of the gyro compass is brought out of its equilibrium position; we assume, e.g., that its north end is given an additional deviation to the east (which corresponds to the position A_1 in Fig.80).

Let us now leave the instrument to itself. We already know (Section 25) that if there were no "eccentric coupling" of the pendulum with the gyro chamber of the instrument then there would be undamped oscillations of the instrument, in which the north end of the instrument axis would describe the elliptical path shown in Fig.80 by the broken line.

At the same time we also know (Section 26) that as a result of the "eccentric coupling" the north end of the gyro-compass axis, being raised above the plane of the horizon, is now given a tendency to sink toward that plane. For this reason, in the presence of "eccentric coupling", the north end of the gyro-compass axis moving toward the west will describe, instead of the upper half of its elliptical path, the lower arc A_1A_2 in Fig.80, after which it will commence its return motion toward the east. Detailed investigation (which we will not discuss here) shows that the successive sweeps of the gyro-compass axis will gradually diminish and that it will gradually approach its equilibrium position N_o, where it will finally stop. The north end of the gyro-compass axis, instead of a closed elliptical path, will describe a spiral curve which will gradually bring it from its initial position A_1 into its equilibrium position N_o.

This constitutes the process of damping the oscillations of the gyro-compass axis due to the "eccentric coupling" of the pendulum with the gyro chamber.

Section 29. Course Deviation of the Gyro Compass.

The gyro compass is designed for installation on seagoing ships. In the preceding sections we did not take into account the influence on the instrument readings exerted by the motions of the ship. The gyro compass is an instrument possess-

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ing extraordinary sensitivity; it is capable of responding to so slow a rotary motion of its base as the diurnal motion of the earth itself. This is precisely why the diurnal rotation of the earth imparts to the instrument the ability to remain in the plane of the meridian. It is obvious that an instrument possessing such sensitivity must very markedly react to any motions of the ship on which it is installed. The errors in the gyro-compass readings due to the motion of the ship on which it is installed are termed its deviations.

At first glance it may seem that there is one case of the motion of a ship which should not be accompanied by any deviations of the gyro compass whatever; this is the case of strictly rectilinear and uniform motion. This is true but it is obvious that there can be no strictly rectilinear motions on the earth's surface at all. Indeed, this is a simple consequence of the spherical shape of the earth. Let us assume that a ship is sailing strictly along the meridian in the direction from south to north (Fig.81). The motion of the ship at a given instant appears to be taking place along the horizontal line ns
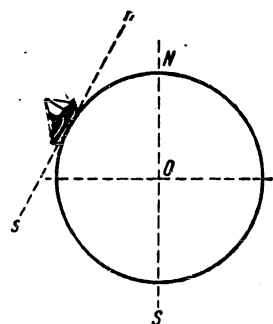
Fig.81

(along the meridian). However, it is obvious that the true path of the ship moving along the surface of the ocean is not the straight line ns but an arc of a circle having a radius equal to the radius of the earth, whose center is located at the center of the earth.

Thus, any motion on the earth's surface which appears to take place along a horizontal straight line is, in fact, a curvilinear motion. For this reason, even in the case when the ship appears to be moving along a straight line at constant velocity, its motion is still accompanied by the appearance of a certain deviation

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of the gyro compass; this deviation is due to the curvature of the earth's surface. Of course, in view of the immense size of the earth[*], the curvature of its surface is very small. Is it possible for such a negligible factor to exert a perceptible influence on the readings of the gyro compass? It has been found that it can. Let us now consider this deviation of the gyro compass, due to the curvature of the earth's surface, during the "rectilinear" and uniform motion of a ship.

Let us assume again that the ship is sailing on the surface of the ocean, maintaining the direction of the meridian from south to north (Fig.82). During a certain time interval, the ship will be displaced from the position M_1 to the position M_2. Let us denote the line so obtained at the point M_1 by $n_1 s_1$, and the corresponding line at the point M_2 by $n_2 s_2$. It is obvious that, on transition of the ship from the position M_1 to the position M_2, the meridian and together with it the plane of the horizon, will have rotated through the angle about an axis perpendicular to the plane of the meridian (Fig.82). Thus, when the ship is displaced along the meridian in a northerly direction, the direction of the plane of the horizon is rotated about a horizontal line normal to the meridian; in this case, the northern part of the plane of the horizon will gradually sink while the southern part rises.
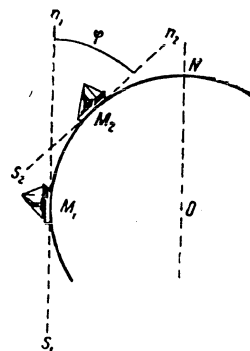
Let us assume now that a gyro compass is installed on the ship and that its axis, at the instant the ship is at position M_1, is directed along the meridian $n_1 s_1$.

Fig.82

* The radius of the earth is 6370 km.

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Since, during motion of the ship in a northerly direction the northern part of the plane of the horizon will gradually sink, it follows that the north end of the axis of the gyro-compass rotor, striving to maintain a constant direction of its axis of rotation, will seem to be rising gradually above the plane of the horizon. On the other hand, we already know (Section 23) that if the north end of the gyro-compass axis rises above the plane of the horizon then the action of the force of gravity of the pendulum will cause the whole instrument to start rotating counterclockwise about the vertical. Consequently, if the gyro compass is installed on a ship sailing in a northerly direction then the influence of the motion of the ship will cause the north end of the gyro-compass axis, which was originally directed along the meridian, to deviate gradually westward from the meridian.

However, when the north end of the gyro-compass axis deviates westward from the plane of the meridian, it will also acquire a tendency to decline toward the horizon; this is a consequence of the rotation of the plane of the horizon about the meridian —let us recall the stars which descend toward the horizon in the western part of the celestial vault. At a certain magnitude of the westerly deviation of the gyro-compass axis from the meridian, the mutually opposing tendencies of the instrument axis to rise and fall due,respectively,to the motion of the ship and the diurnal rotation of the earth, are in equilibrium in this position, the instrument will remain in equilibrium for a long time. This westerly deviation of the instrument axis from the meridian is termed its deviation due to the motion of the ship in a northerly direction.

How extensive is this deviation? At first glance, it may seem that the magnitude of this deviation, which expresses the influence on the instrument reading of so insignificant a factor as the curvature of the earth's surface, should be very small. This is not so. A more detailed examination of the question shows that the magnitude of the deviation with which we are now concerned depends on the speed of the ship and on the latitude of the place. It is found that, at the latitude of

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Leningrad (60°) and at a ship speed of 15 km/hr, which approximately corresponds to a speed of 30 knots, the gyro compass has a deviation of 3°42' and, consequently, reacts markedly to the curvature of the earth's surface; its sensitivity is almost phenomenal.

Thus the motion of the ship in a northerly direction results in a westerly deviation of the gyro compass. It is easy to show that with the ship sailing in the opposite direction the deviation should also reverse its direction. Consequently, the motion of the ship in a southerly direction produces an easterly deviation of the gyro compass.

We have assumed, up to now, that the ship is moving in the direction of the meridian (northward or southward). However, in what way does the motion of the ship in a direction normal to the meridian (eastward or westward) affect the readings of the gyro compass? We have seen that the primary cause of the appearance of deviations of the gyro compass, when the ship is moving in the direction of the meridian, is the rotation of the plane of the horizon about a line normal to the plane of the meridian. Such rotation of the plane of the horizon does not occur if the ship is moving in a direction normal to the meridian. We must conclude that in this case the motion of the ship exerts no influence whatever on the readings of the gyro compass*.

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* This is not entirely accurate. As a result of the diurnal rotation of the earth, the ship is likewise transferred, together with the points of the earth's surface, in a direction perpendicular to the plane of the meridian and toward the east. Consequently, the motion of the ship along the surface of the ocean toward the east or toward the west does exert the same influence on the gyro compass as would be the case if there were a certain increase or decrease in the angular velocity of the diurnal rotation of the earth. This influence is insignificant and may be disregarded.

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Let us now assume that the ship is sailing in some direction, forming a certain angle with the direction of the meridian NS (Fig.83); this angle is termed the course angle or, for short, the course of the ship and is measured in degrees from north to east. Lay off the speed of the ship v in the direction of its motion and resolve it into two components v_1 and v_2, namely the direction along the meridian N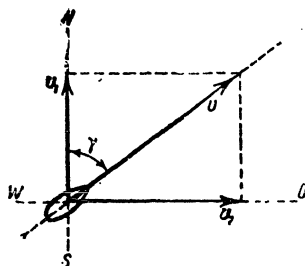S, and the direction perpendicular to it. The component v_2 will have no influence on the gyro-compass readings while the component v_1 produces the deviation discussed above.

Fig.83

Thus, in the motion of the ship in any direction, the deviation of the gyro compass is caused only by the velocity component v_1; the value of this deviation depends on the value of the component v_1 and on the latitude of the place. However, since the value of the velocity component v_1 depends in turn on the value of the velocity v and on the relative bearing or course angle γ then, in the general case, the magnitude of the deviation of the instrument due to the motion of the ship depends on the speed of the ship, on the latitude of the place, and on the course of the ship. For this reason this deviation is termed the course deviation of the gyro compass. In the case when the ship is ascending to more northerly latitudes the course deviation is directed westward; when the ship is descending to a more southerly latitude the course deviation is directed eastward.

We recall that the deviation of damping is always directed eastward (in the northern hemisphere Section 27). Consequently, the course deviation is compounded with deviation of damping during any motion of the ship toward a more southerly latitude and is subtracted from the deviation of damping when it is moving toward northern latitudes. In Section 27, we mentioned a simple device allowing a compensation

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of the error in the gyro-compass readings due to the deviation of damping. In the
same manner, the error due to course deviations which is added (with the plus or
minus sign) to the error due to the deviation of damping is also compensated.

Section 30. The Ballistic Deviation of the Gyro Compass. The Schuler Condition.

In the preceding Section we assumed that the motion of the ship on which the
gyro compass was installed took place at constant speed and with a constant course.
Let us now consider how any variation in the speed or course of the ship will be re-
flected in the readings of the gyro compass.

Let us imagine that we are in a moving streetcar. So long as the car is moving
with constant speed along a rectilinear section of the track, we might entirely fail
to notice the motion of the car if the windows are closed if it were not for the
periodic impacts on the rail joints, to which our attention is involuntarily drawn.
The situation is entirely different when the velocity changes sharply or on a
curved track. At a sudden deceleration, all passengers standing in the car experi-
ence a violent shock which throws them toward the front platform of the car. A
passenger experiences the same shock when the car rounds a curve if the car enters
the curve at a considerable speed; here the direction of the shock is normal to the
direction of motion; if the car turns to the right, all passengers standing in the
car are thrown violently to the left, i.e., toward the side opposite the side on
which the center of curvature of the track is located. Consequently, when the car
moves at variable speed or along a curvilinear section of the track, the passengers
are subjected to the action of a special force which is termed the force of inertia;
if the car is moving along a curvilinear section of the track at constant speed, this
force of inertia is termed centrifugal force.

The same force of inertia is applied to all objects on the ship when it is mov-
ing at variable speed or along a curvilinear path; in particular, the force of iner-
tia is also applied to the pendulum of the gyro compass. No matter how this force

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of inertia J is directed (in all cases, it will be horizontal), it may be resolved into two components J_1 and J_2 directed, respectively, along a north-south line (i.e., along the meridian NS) and along an east-west line (OW in Fig.84).
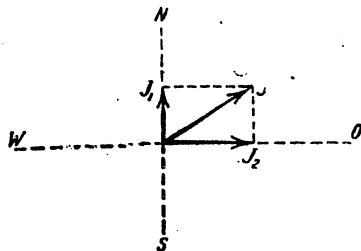

Fig.84

If the gyro-compass axis is directed northward, the pendulum can swing only in the plane of the meridian (Fig. 85). In that case the force J_2, directed perpendicularly to the plane of the meridian, exerts no influence whatever on the pendulum while the force J_1, transmitted to the gyro-compass axis, exerts a pressure F on it which is greater the greater the force J_1. In Fig.85, it is assumed that the force J_1 is directed southward; in that case, the pressure F applied to the north end of the gyro-compass axis will be directed vertically downward. By applying the rule of precession which we already know (Section 6) we conclude that under the action of the pressure F the instrument will precess, rotating counterclockwise about the vertical. Consequently, under the action of the force of inertia J_1, the north end of the gyro-compass axis will deviate from the meridian toward

Fig.85

the west. This deviation of the gyro compass due to the force of inertia of the pendulum when the velocity or course of the ship varies, is termed the ballistic deviation of the gyro compass.

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Let us assume that the ship proceeds at constant speed v along the straight line AB and keeps the course γ_1; then, maintaining this same speed v and describing the arc BC, it goes into the new course γ_2 and continues to move along the straight line CD (Fig.86). How will this maneuver, executed by the ship, affect the readings of the gyro compass?

So long as the ship was moving along the straight line AB, the gyro compass show-ed a constant course deviation corresponding to the velocity v and the course γ_1. On passing along the curve BC, the variable ballistic deviation due to the centrifugal force of the pendu-lum is added to this constant devi-ation. Can we consider that at point C the ship is entering a new rectilinear section of the route having a deviation of the gyro com-pass equal to the course deviation corresponding to the velocity v and the new course γ_2? In other words, can it be considered that at point C the gyro compass will be in a new equilibrium position corresponding to the velocity v and the course γ_2? Of course not. For this reason, on completion of the maneuver, oscillations of the gyro-compass axis must arise which are damped only gradually—under the influence of the "eccentric cou-pling" of the pendulum with the gyro chamber.
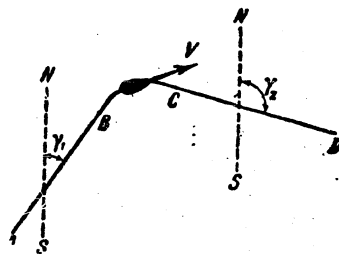
Fig.86

We reach the conclusion that the variation in speed or course of the ship must be accompanied by the generation of oscillations of the gyro-compass axis. It is a remarkable fact, however, that it is possible to design an instrument in such a way as to avoid the generation of such oscillations. For this, it is necessary to ob-serve the following condition: The elements of the instrument must be selected in

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such a way that the period of its undamped oscillations (mentioned in Section 25) is
equal to the period of swing of a pendulum whose length is equal to the radius of
the earth. The period of such a pendulum (i.e., the time of two sweeps) would be
84.4 minutes; this must be the period of the undamped oscillations of the gyro com-
pass to prevent the maneuver of the ship from causing oscillations of the instrument.
This condition, which is often called the Schuler condition from the name of the
scientist who discovered it, is always taken into account in the design of a gyro
compass. However, there are certain difficulties involved in its exact satisfaction,
in view of the fact that the period of the undamped oscillations of a gyro compass
varies with the latitude of the place. At the same time, the reservation must be
made that, even with an exact observation of the Schuler condition, a variation in
the speed of the ship or in its course will fail to result in oscillations of the
gyro-compass axis only where the gyro compass contains no device that might cause
damping of its oscillations, i.e., if there is not "eccentric" coupling of the pendu-
lum with the gyro chamber. For this reason, the maneuver of the ship will still
lead to oscillations of the gyro-compass axis, which will only be damped gradually,
after completion of the maneuver and on passage of the ship to a motion at constant
speed and constant course.

Section 31. The Sperry Gyro Compass with Mercury Reservoirs.

The Sperry pendulum gyro compass, considered in the preceding Sections, was de-
signed around the year 1910 and at that time was widely used on naval ships of
various countries. It was soon found, however, that this instrument had certain
faults; particularly, its readings in a fresh wind proved unreliable. This is due
to the fact that, if the ship rolls, its motion is not strictly rectilinear even if
it maintains a constant course. For this reason the rolling of the ship causes a
deviation of the gyro compass due to the forces of inertia of the pendulum and hav-
ing the character of a ballistic deviation. It must, however, be borne in mind that

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the forces of inertia due to rolling are alternately directed in diametrically op-

posite directions and therefore neutralize each other to a considerable extent. In

spite of this, the rolling still has some influence on the readings of the gyro com-

pass.

The desire to improve the navigational qualities of the gyro compass forced the

American Sperry firm, toward the end of the World War I, to modify the design of the

instrument substantially. The designers of the gyro compass abandoned the use of a

pendulum and replaced this pendulum
by two communicating vessels filled
with mercury. Thus originated a
new type of gyro compass, the Sperry
gyro compass with mercury reservoirs.
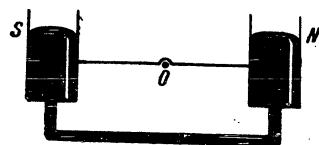
Let us imagine two communicating
vessels S and N, suspended on the

Fig.87

axis O in such a way that the entire system is free to roll in a vertical plane about

the axis O (Fig.87). We assume further that the system of communicating vessels is

balanced in such a way that its common center of gravity coincides with the point O.

In this case, the system of communicating vessels will be in a state of neutral equi-

librium. Let us now fill these reservoirs with a certain quantity of liquid (mercury)

such that its center of gravity will coincide with the point O. Obviously even now

the entire system will be in a state of equilibrium, as long as the vessels S and N

are on the same level (Fig.87).

Now let us change the system of communicating vessels from the position shown in

Fig.87, by raising the vessel N and correspondingly lowering the vessel S (Fig.88).

Now a portion of the mercury will flow from the vessel N into the vessel S, and the

heavier vessel S will thus acquire a tendency to lower still more, while the lighter

vessel N will tend to rise still more. The deviation of the system of liquid-filled

communicating vessels, from the equilibrium position, once it takes place, will have

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a tendency to continue in the same trend. The system of communicating vessels filled with liquid and suspended by this method, has properties similar to those of an "inverted" pendulum, i.e., a pendulum in which the center of gravity is above the point of suspension (Fig.89). Such an "inverted" pendulum, in the vertical position, will be in unstable equilibrium; if ever the smallest deviation from the vertical takes place, this trend will increase continuously. In such an unstable equilibrium are our communicating vessels filled with mercury. These may be termed a "liquid" pendulum. Thus, a "liquid" pendulum of this design resembles in its properties an
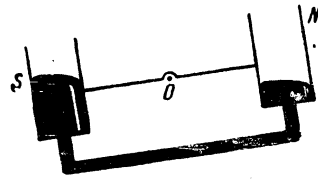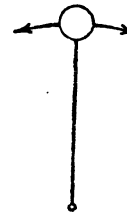
Fig.88   Fig.89

"inverted" unstable pendulum (Fig.89).

It was by such a "liquid" pendulum that the ordinary pendulum was replaced in the new design of the Sperry gyro compass. Let us imagine that communicating vessels, filled with mercury, are attached to the gyro chamber in such a way that one vessel is located on the north side of the gyro chamber (the "north" vessel N), and the other on the south side (the "south" vessel S) (see Fig.90). Let us also assume that the axis of the gyro compass is located horizontally along the meridian; in that case the mercury vessels will be on the same level and, consequently, the entire system will be balanced. We know that the earth's diurnal rotation causes a rotation of the plane of the horizon, together with the meridian, in a counterclockwise direction about the vertical, assuming that the gyro compass is located in the

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northern hemisphere. For this reason, the north end of the gyro-compass axis, hav-
ing a tendency to maintain its constant direction in space under the influence of
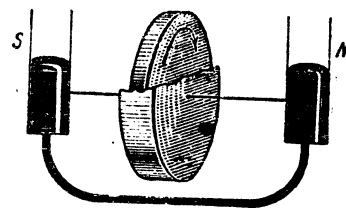the rapid rotation of the rotor,
will deviate eastward from the me-
ridian (Fig.91). However, in that
case, the rotation of the plane of
the horizon about the meridian,
will cause the north end of the
gyro-compass axis and thus also the
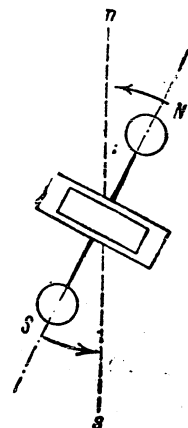northern mercury vessel N, to rise
above the horizon (we recall the

Fig.90

star rising in the east); (see Fig.92). The rise of the vessel N and the drop of the
vessel S cause mercury to flow from the north-
ern vessel N to the southern vessel S. The
southern vessel S begins to be heavier than
the other vessel and this leads to a gener-
ation of the pressures F of the gyro chamber
exerted on the ends of the rotor axis; the
pressure F, directed upward, is applied to
the northern end of the rotor axis, and the
pressure F, directed downward, to the south-
ern end. We now assume that the rotation of
the rotor is clockwise if viewed from the
north-as shown by the curved arrow in Fig.90
and Fig.92. By applying the rule of preces-
sion, we see that the gyroscope begins to

Fig.91

precess, rotating counterclockwise about the vertical, as shown by the arrows in
Fig.92, so that its axis has a tendency to be set in the plane of the meridian.

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As is clear, the gyro compass with mercury vessels possesses the same properties as a gyro compass with pendulum. We remark, however, that in speaking of the gyro compass with pendulum as described in Section 23, this instrument was given a directional force toward the meridian if its rotor was rotating counterclockwise relative to an observer viewing it from the north. We now see that in a gyro compass with mercury vessels, the rotation of the rotor must take place in the opposite sense, i.e., clockwise viewed from the north. Of course, this is connected with
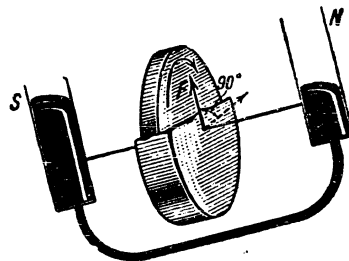

Fig. 92

the fact that the mercury vessels, in their properties, are equivalent to an "inverted" ordinary pendulum.

What then is the advantage of the gyro compass with mercury vessels over the gyro compass with pendulum? The advantage lies in the fact that a gyro compass with mercury vessels provides greater opportunities for adjusting the instrument.

The Sperry gyro compass provides an opportunity for regulating, by special cocks, the quantity of mercury flowing from one vessel to the other when the instrument is tilted. We have seen in Section 25 that the period of undamped oscillations of the compass depends on the latitude of the place. By regulating the quantity of mercury flowing from one vessel to the other, according to the latitude of the place, it is

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possible to make the period of undamped oscillation of the instrument almost inde-

pendent of the latitude of the place, thus making it possible to satisfy the Schuler

condition with great accuracy. In addition, by reducing the diameter of the tube

through which the mercury flows from one vessel into the other, a certain lag in the

flow of mercury can be introduced which in turn improves the operation of the

instrument during a roll. The obvious advantages of a gyro compass with mercury

vessels have led to a rapid displacement of the pendulum gyro compass in world prac-

tice by this new type of Sperry instrument.

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CHAPTER IV

THE GYRO HORIZON AND GYRO VERTICAL

Section 32.   The Gyro Pendulum

The direction of the vertical or the plumb line, at a given place, is determined very simply by means of the ordinary plumb line, that is, a string held at one end with a weight at the other end. The direction of this string under equilibrium condition gives the direction of the vertical at a given place. Peculiar difficulties arise when the direction of the vertical must be determined on some moving object, such as a ship or aircraft; these difficulties are due to the fact that an ordinary pendulum, one type of which is the plumb line, does not possess a sufficient degree of stability. The effort to increase this stability naturally leads to the thought of replacing the simple weight in the pendulum by a rapidly rotating rotor. In this way, we arrive at the idea of a gyro pendulum.

Let us imagine a pendulum swinging in a vertical plane, alternately rotating about the fixed horizontal axis AA, now to one side and now to the other, and consisting of the rod A with the ring B at its end. To this ring the ends of the axis of rotation CC of the small top or rotor C are attached (Fig.93). Let us put the top C in rapid rotation about the axis cc, and then, by a light tap, let us bring our pendulum out of the vertical equilibrium position. It will then begin to swing about the axis aa. The question now arises how the rotation communicated to the top C is reflected in these swings. Does it make the vertical position of the pendulum more stable? Does the rotation of the top impart to the pendulum the ability of

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resisting a force attempting to move it from the vertical position?

We have seen, in Sections 7 and 9, that a rapid rotation gives stability to the axis of a gyroscope having three degrees of freedom but that a gyroscope with two degrees of freedom does not possess this property in the slightest degree. Since the gyro pendulum shown in Fig.93 has two degrees of freedom (corresponding to the two rotations possible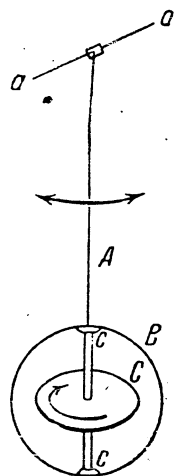 in this system, about the axes aa and cc, respectively), we must conclude that any rotation of the rotor of the pendulum of this device, no matter how fast, will not in the slightest degree increase the stability of the vertical position of that pendulum. It is easy to verify by direct experiment that the rotation of the rotor C does not at all influence the swinging of the pendulum. In order to detect the stabilizing influence of the rotation of the rotor on the swinging of the pendulum, it is necessary to impart to the instrument the third degree of freedom which it now lacks.

This can be done by attaching the rod A of the pendulum to the inner ring of the Cardanic suspension, both rings of which are rotating about the mutually perpendicular and, in the normal position, horizontal axes aa and bb (Fig.94). Now the instrument has three degrees of freedom corresponding to the three rotations about the axes aa, bb, and cc. The same result may be reached by resting the instrument on the horizontal stage L by means of the point shown in Fig.95; here too, the pendulum may swing, deviating from the vertical in more than a single plane, i.e., performing not only plane but also spatial oscillations.

Fig.93

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In a gyro compass with three degrees of freedom (Fig.94 or Fig.95) the stabiliz-
ing influence of the rotation of the rotor on the swinging of the pendulum is clear-
ly detected. Let us consider the motion of such a gyro pendulum with three degrees
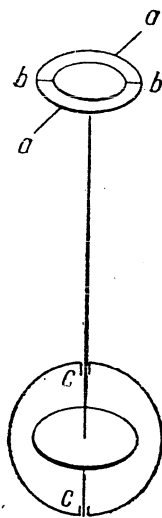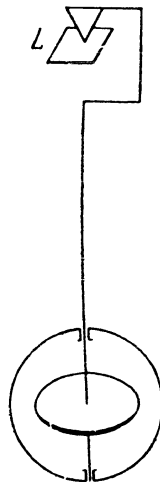of freedom.
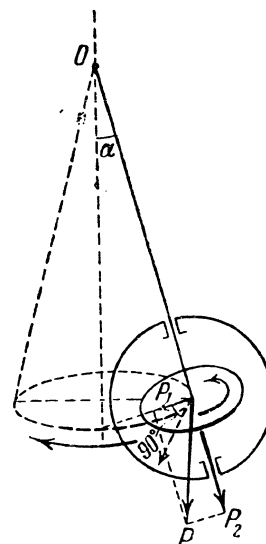

Fig.94   Fig.95   Fig.96

Let us give the rotor of the pendulum a rapid rotation (for example, counter-
clockwise, viewed from above) and let us then cause it to deviate by a certain angle
from the vertical (Fig.96). Here, the letter O denotes the point of intersec-
tion of the Cardanic axes, which remains motionless, in the gyro pendulum design of
Fig.94, or the end of the base point in the design of Fig.95. If we now release our
gyro pendulum it will not, by any means, begin to swing in the same vertical plane
which it occupied when it was made to deviate from the vertical.

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Let us now proceed, as explained in Section 6 when we discussed the action on the gyroscope of a force applied at a point of its axis and directed along a line not normal to the axis. At the center of gravity of the gyro compass (which we assume to lie on the axis of the rotor) is applied the weight of the gyro pendulum P. Let us resolve the force P into two components P_1 and P_2, of which P_2 is directed along the rotor axis of the gyro pendulum and P_1 is perpendicular to that axis. The component P_2 is balanced by the resistance of the fixed point 0, but the component P_1 causes a precessional motion of the gyro pendulum about the vertical passing through the fixed point 0. The sense of this precessional rotation is easily found by the rule of precession, which we already know.

By applying this rule (Fig.96) we see that, in this case, the gyro pendulum will describe a cone rotating counterclockwise (when viewed from above) about the vertical.

In Section 10, we described an experiment with a gyro made of a bicycle wheel (Fig.18). In essence, the model of the gyro then discussed was nothing but a gyro pendulum and the precessional rotation of the instrument described in Section 10 differs in no way from the phenomenon of which we are now speaking. We remark merely that, in Section 10, we assumed the gyro axis to be horizontal, i.e., normal to the vertical axis of precession while here we are considering the more general case of a gyro compass deviating by an arbitrary angle from the vertical.

In Section 10, in speaking of the experiment with the bicycle wheel, we remarked that the higher the angular velocity of the proper rotation of the wheel, the lower will be the angular velocity of the precessional motion. This remark still remains valid in the more general case now under discussion. The more rapidly the rotor of a gyro pendulum rotates about its axis, the more slowly does the precessional motion of the gyro pendulum about the vertical take place.

Let us assume again that the gyro pendulum is in its vertical equilibrium position and that its rotor is placed in rapid rotation (Fig.97). Let us now strike

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the gyro compass by hand at its lowest point A, in a direction perpendicular to the plane of the paper. How will the pendulum react to this impact?

The effect of the shock will be expressed in the application, during the extremely short time of the impact γ to the point A of the gyro-pendulum, of the impact force S in the direction of the shock, i.e., in a direction normal to the plane of the paper (Fig.97). Since the force S is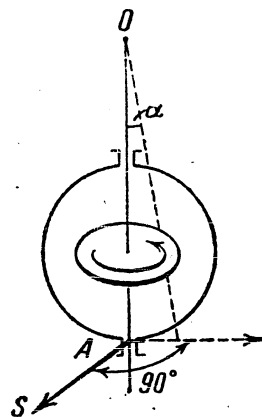 applied at a point of the gyro rotor axis, and since it is directed normal to this axis, the result of the action of the force S is found by the rule of precession (Fig.97). By applying this rule (as before, we assume the rotation of the gyro rotor to be counter-clockwise if viewed from above), we note that as a result of the shock, the gyro pendulum deviates from the vertical in a plane normal to the direction of the shock (more specifically, to the right) by a certain angle , which is found from eq.(3) of Section 6:

$$\mathcal{L} = \frac{Sa\,\gamma}{J\,w}$$

Fig.97

(where a = OA is the distance between the point of application of the force S and the fixed point O; J is the moment of inertia of the rotor; and w is the angular velocity of its proper rotation).

In deviating, during the time τ, by this angle α, the gyro pendulum begins to precess about the vertical under the action of the force of gravity, describing a cone about the vertical, as explained above.

It is clear from eq.(3) that the angle α by which the gyro pendulum deviates from the vertical under the action of the shock will be smaller, the higher the angular velocity w of the proper rotation of its rotor. It is in the smallness of

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this angle of deviation α that the stability, communicated to the gyro pendulum by the rapid rotation of its rotor, is expressed.

Section 33. The Fleuriais Marine Gyro Horizon

We have an example of the practical employment of the gyro pendulum in the Fleuriais marine gyro horizon.

In Section 1 it was mentioned that the first attempt to utilize the properties of a rapidly rotating top in practice was made by Serson as early as the Eighteenth Century but it ended with the failure of the attempt to design a gyroscopic "artificial horizon" which would have replaced, in foggy weather, the visible horizon required by the navigator for astronomic observations. The original idea of Serson was put into practice only at the end of the Nineteenth Century in the instrument of the French inventor Fleuriais.

The Fleuriais marine gyro horizon is an additional attachment to the ordinary sextant, which measures the altitude of a heavenly body above the horizon. In t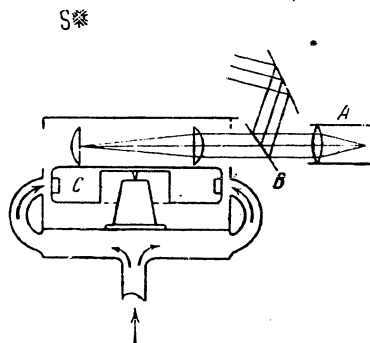he sextant, the star S and the horizon line are simultaneously viewed through the sight tube A (Fig.98). In the Fleuriais instrument an artificial gyro horizon is placed in front of the mirror B. It consists of the rotor C, which rests on a point somewhat above the center of gravity. Before beginning the observation, the rotor is spun by compressed air from a hand pump. The jet compressed air enters the instrument through a hollow shaft and impinges on depressions in the lateral surface of the rotor; during the time of

Fig.98

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observation, the rotor is spinning at an angular velocity of 50 revolutions per second. Since the center of gravity of the rotor is lower than the end of its supporting point, we have here a gyro pendulum and, what is more, one with three degrees of freedom.

On the upper surface of the rotor, each end of one diameter has a plane-convex lens; on the flat surface of each lens is drawn, at the height of the optical axis, a fine line perpendicular to the rotor axis. The distance between the lenses is equal to their focal length; for this reason, the line drawn on each of the lenses is clearly visible in the tube A, at each revolution of the rotor.

If the axis of the rotor is strictly vertical then during each revolution one line or the other appears to be strictly horizontal and at the same level to the observer looking into the tube A. All successive images of the lines are merged in the observer's eye into a single horizontal line, which is able to replace the line of the natural horizon invisible in foggy weather.

However, if the rotor axis deviates from the vertical by even an insignificant angle, a relatively slow precessional rotation of the rotor about the vertical begins immediately, in the course of which the rotor axis will describe the surface of a cone about the vertical. To the observer, looking into the tube A, the line visible in the tube, replacing the line of the natural horizon, appears to be continuously changing its position. In Fleuriais instrument, the period of precession (i. e., the time of a single rotation of the rotor axis about the vertical) is equal to approximately two minutes. During these two minutes, the line visible in the tube appears to be horizontal twice, at its highest and lowest positions; in all intermediate positions it appears to be inclined, now to one side and now to the other. This mobility of the artificial horizon does not interfere with the observations; the technique of astronomic observation developed with the Fleuriais instrument takes into account the characteristic oscillations of the artificial gyro horizon.

Section 34. The Pendulum Aircraft Course Corrector

If the center of gravity of a gyro pendulum coincides with its fixed point, this instrument is converted into an astatic gyro and becomes unable to align its axis with the vertical. In the Sperry gyro horizon, which will be discussed in the following Section, a very interesting method of aligning the axis of the astatic gyro with the vertical is used. In the present Section the method of Sperry, which may be termed that of the "air pendulum correction", will be discussed.

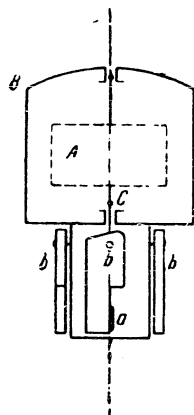The rotor of the astatic gyro A is suspended in a Cardanic suspension, whose inner ring is designed as the gyro chamber B (Fig.99); the outer ring is not shown in the diagram; the point of intersection of the Cardanic axis coincides with the center of gravity C of the entire system (the axes of rotation of the chamber and of the outer ring are horizontal). The gyro chamber B has a rounded cross section in its upper part and a square cross section in its lower part. The four walls of the lower part of its chamber are provided with similar and symmetrically arranged windows a, which are partly covered by the pendulum shutters b.

In the gyro chamber B an elevated air pressure is maintained (in the following Section we will explain how this is accomplished in the Sperry instrument). If the gyro axis is vertical, as assumed in Fig. 59, all four windows a will be half covered and permit ejection of the same jets of compressed air from the chamber. Each jet exerts a reactive pressure in the opposite direction on the chamber. Since all these pressures are equal and are directed in pairs at opposite sides, they are in mutual balance, so that the gyro axis remains

Fig.99

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in a vertical position.

Let us now assume that the gyro axis deviates from the vertical by a certain angle in a plane parallel to two opposite faces of the lower part of the gyro chamber (Fig.100). One of the two opposite windows a in these faces will then be completely open and the other (not shown in Fig.100) will be closed. The reactive pressure caused by the air jet enteri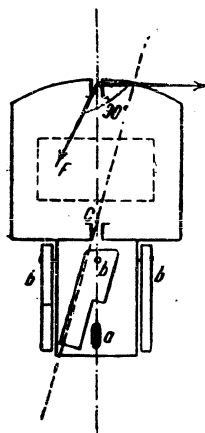ng through the open orifice in the front face will no longer be balanced by the opposite reactive pressure, since the orifice in the back face is closed; as a result, a reactive pressure directed perpendicularly to the plane of the paper, in the direction away from the reader[*], will act on the lower part of the chamber. This reactive pressure, transmitted to the gyro axis, will produce the force F applied at the upper end at this axis and directed normal to the plane of the paper in the direction toward the reader (Fig.100).

Let us now assume that the rotor of the gyroscope rotates counterclockwise, if viewed from above (Fig.101). By using the rule of precession, we conclude that under the action of the force F the gyro axis will precess in the vertical plane and will

Fig.100

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[*] At a very small angle of deviation of the gyro axis from the vertical, both these orifices will remain open, but one orifice will be open more than half and the other less than half. The reactive pressures corresponding to the opposing air jets will not be balanced; as a result, even in this case, there is a resultant reactive pressure on the chamber, directed normal to the plane of the paper and away from the reader.

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approach the vertical until it coincides with it. In this way, the original devi-

ation of the gyro axis from the vertical will be eliminated.

We have assumed that the gyro axis originally deviated from the vertical in a

plane parallel to two opposite faces of the lower part of the chamber. However,

since any deviation of the gyro axis from the vertical can be resolved into two such

deviations, what has been said now still remains true even in the general case of
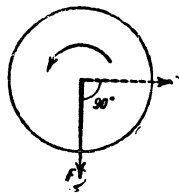
any deviation of the gyro axis from the verti-
cal. Thus, the pendulum air correction in
all cases brings the axis of the astatic gyro
into a vertical position.

In our discussion we have assumed that
the astatic gyro with "pendulum air correc-
tion" is set up on a fixed base. The situa-
tion becomes more complicated if such a gyro

Fig.101

is set up on a moving base, e.g., in the cabin of an aircraft in flight.

If the aircraft is flying with a certain acceleration, then all objects on that

aircraft are subjected, in addition to the force of gravity, to a corresponding

force of inertia, having a direction opposite to that of the acceleration of the

aircraft. In this case, the ordinary pendulum suspended in the aircraft cabin, in

its equilibrium position, is located not along the true vertical but along what is

called the "apparent" vertical, i.e., in the direction of the resultant of the force

of gravity and the force of inertia. Consequently, the pendulum shutters, in their

equilibrium positions, are also located along the "apparent" vertical rather than

along the true vertical.

The reactive pressures of the air jets issuing from the windows a are mutually

balanced in the case when all the windows are half covered by the shutters b. This

will take place when all the shutters b are parallel to the axis of the instrument,

i.e., to the axis of the gyro. It follows from this that, in the equilibrium

position of the instrument, the gyro axis is likewise located not along the true
vertical but along the "apparent" vertical; in this way, when the aircraft moves
with an acceleration, the shutters of the pendulum correction, which themselves
deviate from the true vertical, will entrain the gyro axis; this will force the gyro
axis to precess slowly in the direction not of the true vertical but of the "apparent" vertical.

If we take into consideration the fact that, during the flight of an aircraft,
its acceleration and consequently, also the corresponding forces of inertia, are
continuously varying in both magnitude and direction, we are able to say that in the
aircraft cabin, a gyro with a pendulum air correction will be, as it were, a means
of averaging the equilibrium positions of the ordinary pendulum and a damper on
their oscillations.

Section 35. The Sperry Aircraft Gyro Horizon

We mentioned in the preceding Section that the method of the pendulum air correction was used by Sperry in his design of the aircraft gyro horizon. This is one
of the blind flying instruments. It makes it possible for the pilot, when the natural horizon is invisible, to detect any deviation from horizontal flight (diving or
climbing of the aircraft) as well as any banking.

An astatic gyro with a pendulum air correction is placed in the hermetically
sealed case of the instrument A which is attached to the instrument board of the
aircraft in front of the pilot (Fig.102); the direction of flight is indicated on
the diagram by the arrow. The inner ring of the Cardanic suspension is designed as
the gyro chamber B, and the outer ring has the form of the rectangular frame C. The
frame C is coupled to the joint D, about which the bent lever DEF can rotate. The
part EF of this bent lever is visible to the pilot through the glass G in the form
of a white bar; this bar plays the role of the horizon line in the instrument. Beyond this bar, the pilot also sees, through the glass G, the cylindrical surface H

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which is painted in two colors; the light blue color of the upper part of this back-
ground merges into the dark-gray of the lower part, corresponding to the color of
the sky and the earth, respectively.

In the normal position of the instrument, both Cardanic axes are horizontal:
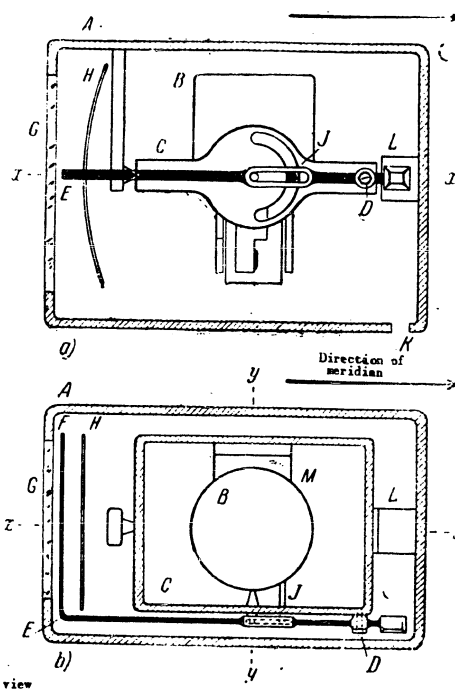

a)   Direction of
                      meridian

b)
Side view

Fig. 102

the axis of rotation xx of the outer frame C, along the longitudinal axis of the
aircraft and the axis of rotation yy of the gyro chamber, transverse to the aircraft.
The bent lever DEF is coupled with the gyro chamber by means of the pin J, which is
attached to the gyro chamber and passes through an arc-shaped slot in the part DG of

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the outer frame and through a rectilinear slot in the part DE of the bent lever.

In its normal position, the bent lever DEF is horizontal.

In the case of the instrument A, a vacuum is produced by the continuous aspira-tion of air by means of a Venturi tube through the orifice K. On the other hand, the inner cavity of the gyro chamber is connected with the atmosphere by means of a channel in the outer frame C, in the bearing L of the x-axis of the outer frame C and in the bearing M of the y-axis of the chamber B. In this way, the aspiration of air from the inner shell A produces a continuous air stream: the outer air, entering the gyro chamber B through the above-mentioned channel, is ejected into the cavity of the inner shell A through the window of the lower part of the chamber and is then again aspirated by a Venturi tube. The air jet entering the gyro chamber B impinges on the depressions in the gyro rotor, and thereby maintains it in rapid rotation; then, in leaving through the window of the lower part of the gyro chamber, it ensures proper functioning of the pendulum air correction. As a result of this, during the time of flight, the gyro axis maintains its vertical alignment at any position of the instrument.

Since, during the operation of the instrument, the gyro axis preserves its verti-cal position, the axis yy of the gyro chamber B, which is perpendicular to it, like-wise stably remains in the horizontal direction. The part EF of the bent lever DEF is parallel to the axis yy. Consequently this part of the bent lever is also visi-ble to the pilot through the glass, playing the role of a "horizon bar" in the in-strument and, in its function, actually maintaining the horizontal position.

If the airplane is in rectilinear horizontal flight and is not banking, then the instrument case, attached to the instrument panel of the aircraft, occupies the position shown in Fig.102. The bent lever DEF likewise occupies the position in Fig.102. The "horizon line" EF is visible to the pilot in the center of the glass G; likewise at the level of its center, shows a miniature silhouette of the aircraft (flying in the direction away from the observer). Consequently, during horizontal

unbanked flight, the silhouette of the airplane on the glass of the instrument is
represented as projected on the "horizon line" EF (Fig.103).

Let us now imagine that the airplane is descending along an inclined straight
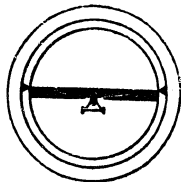line (or, as they say, the airplane is diving).
The instrument case A, rigidly attached to the
instrument board of the aircraft, now occupies the
inclined position shown in Fig.104; the gyro cham-
ber A maintains its former position, since the
gyro axis, as we know, remains vertical; the bent
lever with the "horizon line" EF then occupies the
position shown in Fig.104. It is clear that the "horizon line", remaining horizon-
tal, is now shifted toward the top of the glass G. Consequently, the miniature
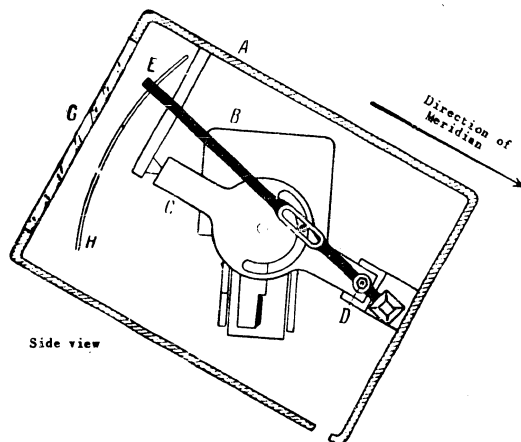
Fig.103

Fig.104

silhouette of the aircraft imaged on the glass G will now appear to the pilot as
projected against the colored background below the "horizon line" EF (Fig.105).

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It is easy to understand that, during flight along an inclined straight line upward (climbing), the "horizon line" will shift toward the bottom of the glass G, and the silhouette of the aircraft will be projected on the colored background above the "horizon line". In this way, the pilot will be able to determine, from the indications of the instrument, whether the flight is horizontal or not.

In exactly the same way, any banking by the aircraft can be determined from the instrument. If the aircraft is flying with a bank, then the miniature silhouette
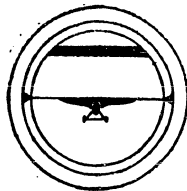
Fig.105   Fig.106
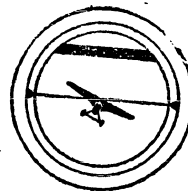
of the aircraft imaged on the glass of the instrument will likewise go into such a bank, while the "horizon line", as we know, will always remain horizontal. If the silhouette of the aircraft on the instrument appears to the pilot to be located below the "horizon line" and in addition, to have a right bank (Fig.106), then in reality the airplane is descending with a right bank.

The remarks made at the end of Section 34 as to the errors introduced in the operation of the air pendulum correction by the accelerations in the motions of the instrument base, must of course also be applied to the Sperry gyro horizon. The accelerations in the motions of the aircraft result in corresponding errors in the instrument readings. When an aircraft is flying with a certain acceleration, the "horizon line" in the instrument ceases to be strictly horizontal. However, since at not too great an acceleration, the deviation of the "apparent" vertical from the

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true vertical remains small, it follows that the deviation of the "horizon line"
in the instrument from the horizontal is likewise small.

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